An Outline of the New Keynesian DSGE

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1 Overview

The micro-foundations of an NK DSGE are twofold. On the side of the household, a representative agent maximizes life-long expected utility by deciding the flow of consumption and labor supply. The optimal conditions include one intertemporal and the other intra-temporal. The former refers to the typical Euler equation. The latter refers to the optimal allocation between consumption and leisure(or labor supply) within each period. On the firms' side, individual firms maximize expected discounted profits by setting the price. The forward-looking price-setting takes into account price stickiness. Specifically, it reflects the considerations of expected inflation and future changes in marginal costs. Since the firms are monopolistic, once the price set, the demand for each good, the production and labor demand are correspondingly determined.

Finally, we impose clearing in each good market and labor market. A monetary policy rule is given. These together give a well defined dynamic system of equilibrium.

2 Model

2.1 Household

Utility function takes the following form, where σ is the coefficient of relative risk aversion, and its inverse is the elasticity of intertemporal substitution. $\frac{1}{\phi}$ is the Frisch elasticity of labor supply, which captures the pure substitution effect of wage on labor supply taking income effect as given.

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi}$$

The consumer's optimization problem is

$$\begin{aligned} \underset{\{C_t, N_t\}}{\text{Max}} \quad & \sum_{t=0}^{+\infty} E_0[\beta^t U(C_t, N_t)] \\ s.t. \quad & P_t C_t + Q_t B_t = B_{t+1} + W_t N_t + \tau_t \end{aligned} \tag{1}$$

Q_t is the price of a bond that pays 1 at time $t+1$. It is the inverse of gross nominal rate. $Q_t = \frac{1}{1+i_t}$.

Remember C_t is a consumption bundle across all goods.

$$C_t = [\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di]^{\frac{1}{\epsilon-1}} \tag{2}$$

Inside the consumption bundle, optimal allocation requires that consumer's demand for each good i is a fraction of the total demand, where the fraction depends on relative price.

$$C_t(i) = (\frac{p_t(i)}{P_t})^{-\epsilon} C_t \tag{3}$$

We study aggregate price behavior. It can be also written in a CES form.

$$P_t = [\int_0^1 P_t(i)^{1-\epsilon} di]^{\frac{1}{1-\epsilon}} \tag{4}$$

Lastly, there is a transversality condition(TVC) that prevents Ponzi schemes.

$$lim_{\{T \to +\infty\}} E_t(B_T) \geq 0 \tag{5}$$

2.2 Firms

Monopolistic firms produce a unit continuum of goods indexed by i.

$$Y(i)_t = A_t N(i)_t^{1-\alpha} \tag{6}$$

Individual goods are aggregated into total output with the CES function form. ϵ is the elasticity of substitution.

$$Y = (\int_0^1 Y(i)^{1-\frac{1}{\epsilon}} di)^{\frac{\epsilon}{1-\epsilon}} \tag{7}$$

Monopolistic firm i sets the price at the time t to maximize expected discounted profits.

In each period, the probability of being able to reset the price is θ. Therefore, the firm's revenue in the case of non-resetting in the future period is still computed with the price set at t. Also, the demand for good depends on that price. In the same time, the total cost $\Psi_t(Y_{t+s|t})$ faced by firm i at t is a function of the demand given price set by t. Thus it varies from period to period.

$$Max_{\{p^*(i)\}} \quad \sum_{s=0}^{+\infty} \beta^s \theta^s E_t(p^*(i) Y(i)_{t+s|t} - \Psi_t(Y_{t+s|t})) \tag{8}$$

Notice the term $(1-\theta)^s$ is the probability that by time $t+s$, the firm remains unable to reset the price since time t. $Y(i)_{t+s|t}$ represents the demand for good i at time $t+s$ given the price set at time t. $N(i)_{t+s}$ is then trivially determined via production function. Wage is determined by the labor market clearing condition.

2.3 Aggregate Price Dynamics

Omitting the details of Calvo pricing, the aggregate price change is:

$$\pi_t = (1-\theta)(P_t^* - P_{t-1}) \tag{9}$$

P_t^* is the optimal price set by firms at time t. Only a $1-\theta$ fraction of the firms is able to reset the price. Therefore, aggregate price changes are proportional to $1-\theta$.

3 Optimal Conditions and Equilibrium

3.1 Households

The F.O.C. of consumer's problem

Intertemporal Condition:

$$Q_t = \frac{1}{1+i_t} = \beta E_t((\frac{C_{t+1}}{C_t})^{-\sigma}\frac{P_t}{P_{t+1}}) \tag{10}$$

Define $\beta = \frac{1}{1-\rho}$, ρ is the time discount rate. The log-linearized version of Equation 10 is

$$c_t = E_t(c_{t+1}) - \frac{1}{\sigma}(i_t - E_t(\pi_{t+1} + \rho)) \tag{11}$$

Intratemporal Condition:

$$\frac{W_t}{P_t} = C_t^{\sigma} N_t^{\phi} \tag{12}$$

To log-linearize it

$$w_t - p_t = \sigma c_t + \phi n_t \tag{13}$$

3.2 Firms

The optimal price setting of individual firms leads to the following behavior of aggregate inflation. It is log-linearized around zero inflation.

$$p_t^* - p_{t-1} = (1-\beta\theta)\sum \beta\theta^s E_t(\widehat{mc}_{t+s|t} + p_{t+s} - p_{t-1}) \tag{14}$$

$\widehat{mc}_{t+s|t}$ is the deviation of marginal cost from its flexible regime level. The optimal price change is proportional to expected deviations in marginal cost and changes in overall inflation over the future periods.

The deviation of marginal cost from a flexible price regime is proportional to the output gap. When output is high, the marginal cost is high as the labor demand drives up the wage.

$$\widehat{mc}_t = (\sigma + \frac{\phi+\alpha}{1-\alpha})(y_t - y_t^n) \tag{15}$$

In the meantime, each firm's labor demand simply satisfies the desirable level of production.

$$N_t(i) = (\frac{Y_t(i)}{A_t})^{\frac{1}{1-\alpha}} \tag{16}$$

3.3 Market Clearing

Each good market clears.

$$Y_t(i) = C_t(i) \quad \forall i \tag{17}$$

The labor market also clears.

$$\int_0^1 Y_t(i) di = N_t \tag{18}$$

Combining it with Equation 16, 17, 3, take integral over i and log-linearze it.

$$(1-\alpha)n_t = y_t - a_t + d_t \tag{19}$$

d_t is a measure price dispersion. Around zero inflation, d_t is approximated to be zero. Any deviation from zero inflation implies positive dispersion.

4 Summary of the Key Forces

Equation 14,15 together gives New Keyesian Phillips Curve(NKPC). It speaks to a positive correlation between output gap and inflation.

Equation 11 and 17 together gives an dynamic IS curve, or Euler equation. It governs how the short-run demand is determined real interest rate prevailing in the current period. The real rate is altogether determined by the nominal rate set by the central bank and the expected inflation.

In a forward-looking NK model, the expectation of inflation is implicitly pinned down by an interest rate rule.

Here is a summary of the system.

$$DIS: \quad \tilde{y}_t = E_t \tilde{y}_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - r_t^n) \tag{20}$$

$$\begin{gathered} NKPC: \quad \pi_t = \beta E_t(\pi_{t+1}) + \kappa \tilde{y}_t \\ NaturalInterestRate: \quad r_t^n = \rho + E_t(\Delta y_{t+1}^n) \end{gathered} \tag{21}$$

$$MonetaryPolicyRule: \quad i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t \tag{22}$$

It is important to know that for inflation to be stable, interest rate rule needs to be responsive enough to inflation and output. Specifically, it requires

$$\kappa(\phi_\pi - 1) + (1-\beta)\phi_y > 0$$