Perceived versus Calibrated Income Risks in Heterogeneous-agent Consumption Models

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Roadmap

Motivation

Empirical Evidence

- Framework
- Perceived v.s. calibrated risks
- Unemployment risks
- Perceived risks and decisions

Model

- Objective model
- Subjective model

Conclusion

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- Risks matter for individual decisions
  - precautionary saving
  - stock market participation
  - portfolio choice

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    - $\rightarrow$ heterogeneous MPCs
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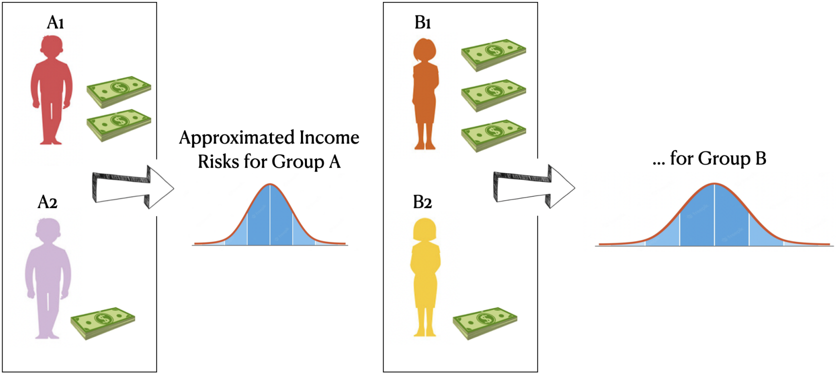
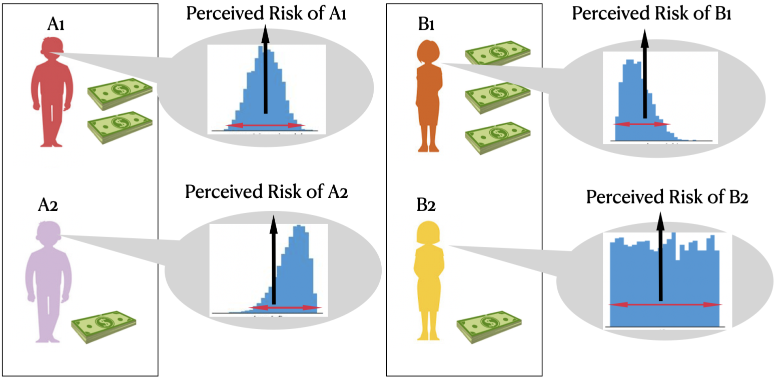
    - $\rightarrow$ business cycle fluctuations
- Income risks are central inputs of any incomplete-market model
  - Conventional approach: calibrated risk from panel data
  - This paper: directly perceived risks from a survey

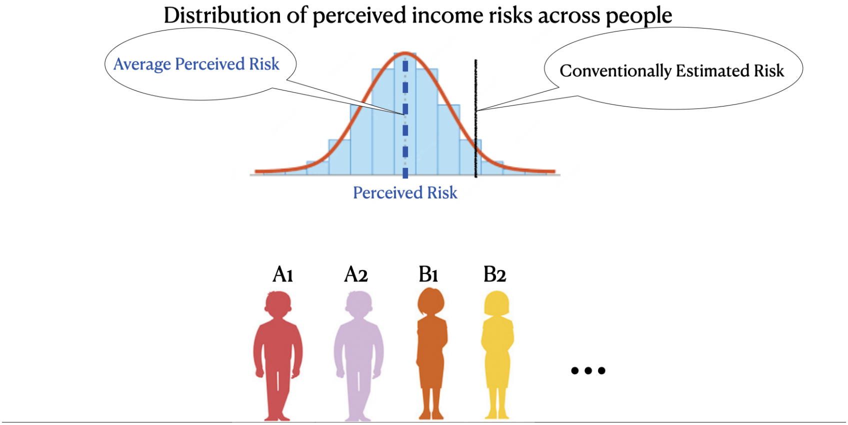
Conventional calibration: estimated from panel data



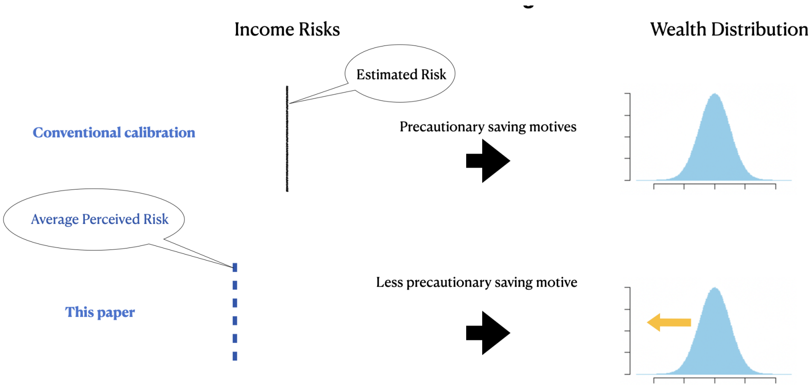
This paper: reported perceived risks in a survey



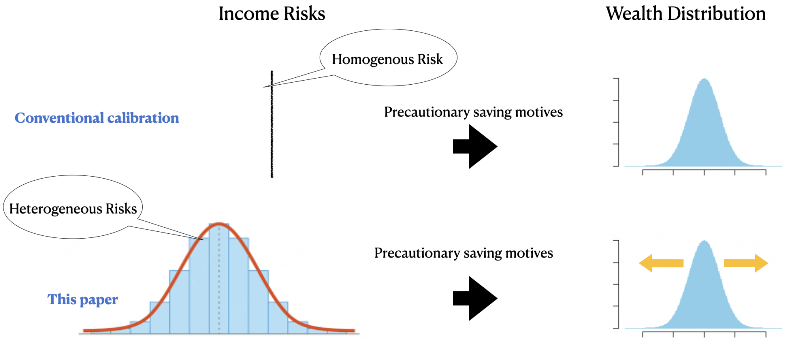
Perceived versus Calibrated Risk



Smaller *perceived* risks → lower level of savings



Heterogeneous risks → differential savings



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- Wage growth

$$\Delta w_{i,t+1} = \Delta z_{i,t+1} + \Delta e_{i,t+1}$$

- individual i at time t
- the time-series nature of $e_{i,t}$ to be specified later

Perceived risks (PR) versus calibrated risks

- To the agent: **conditional** variance under FIRE

$$Var^*_{i,t}(\Delta w_{i,t+1}) = Var^*_{i,t}(\Delta e_{i,t+1})$$

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$$Var^*_{i,t}(\Delta w_{i,t+1}) = Var^*_{i,t}(\Delta e_{i,t+1})$$

- To econometricians: **approximated unconditional** variance

$$Var_c(\Delta \hat{e}_{i,c,t+1}) = Var_c(\Delta w_{i,t+1} - \Delta \hat{z}_{i,t+1})$$

- $\hat{e}_{i,c,t+1}$: the first-step regression residual controlling observable vars
- group c: **assumed** to share income process/risks
    - e.g. education/year of birth/gender/age

Limitations with risk estimates from panel data

- Superior information/unobservable heterogeneity: $\hat{z}_{i,t} \neq z_{i,t}$
  - $\hat{z}_{i,t}$ unlikely capture all in the information set of i at t
    1. Intrinsic heterogeneity of individual i
    2. Foresight about individual circumstances

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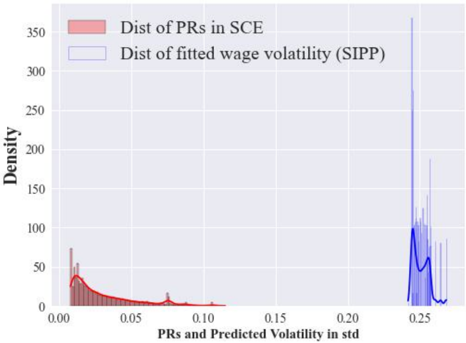
- Model misspecfication
  - Risks may differ within group c
- Surveyed PR can be a useful alternative
  - Directly conditional on information set of each i at t
  - No need to restrict risk heterogeneity by group c
  - Drives behaviors even if they are subjective

Perceived risk v.s. wage volatility

Conditional v.s. unconditional



- PR < wage volatility
- PRs are more heterogeneous than the dispersion of wage volatility explained by observable factors

Time series structure of wage shocks

$$e_{i,t} = \underbrace{p_{i,t}}_{\text{permanent}} + \underbrace{\theta_{i,t}}_{\text{transitory}}$$

$$p_{i,t} = p_{i,t-1} + \psi_{i,t}$$

$$\psi_{i,t} \sim N(0, \sigma^2_{i,t,\psi}), \quad \theta_{i,t} \sim N(0, \sigma^2_{i,t,\theta})$$

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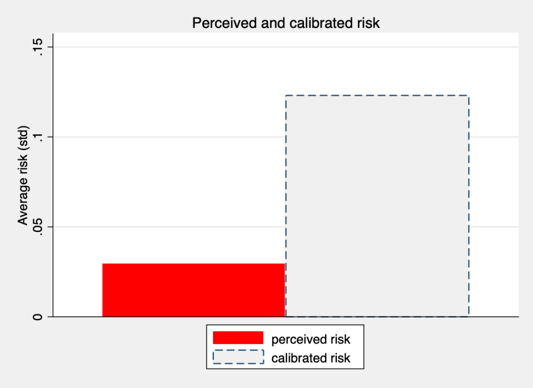
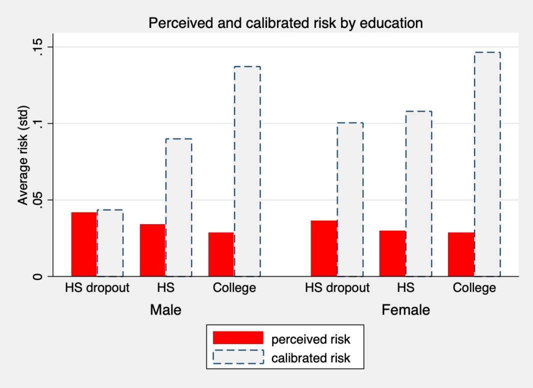
- The agent's PR: $Var^*_{i,t}(\Delta w_{i,t+1}) = \sigma^2_{i,t+1,\psi} + \sigma^2_{i,t+1,\theta}$
- Econometricians' calibrated risk

$$\widehat{Var}_{c,t}(\Delta \hat{e}_{i,c,t+1}) = \hat{\sigma}^2_{c,t+1,\psi} + \hat{\sigma}^2_{c,t+1,\theta}$$

Average PR < calibrated risk



PRs < calibrated risks **within groups**



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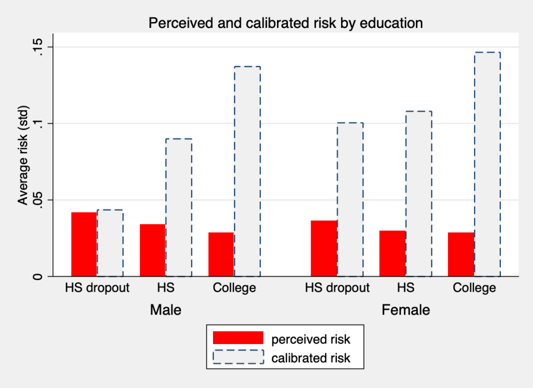
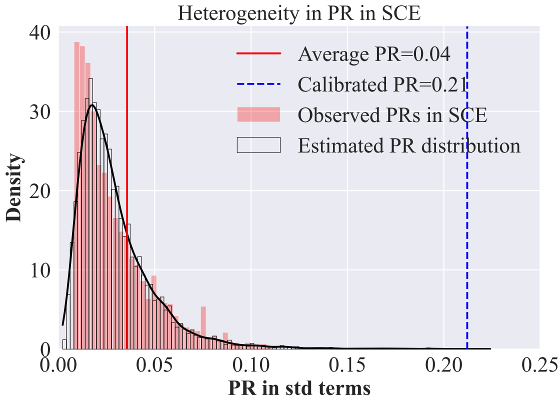


- The wage risk estimates by Low, Meghir, and Pistaferri, 2010:
  - low education: permanent risk = 0.09, transitory risk = 0.08
  - high education: permanent risk = 0.106, transitory risk = 0.08

What explains the PR heterogeneity?

- Observables + time FE: $R^2 = 0.10$
- Individual fixed-effects only: $R^2 = 0.60$

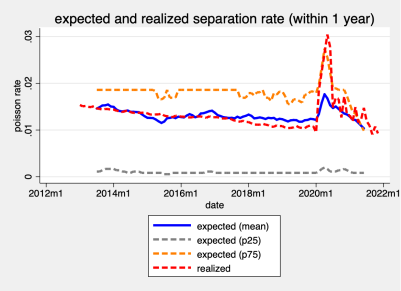
Accounting for the survey evidence

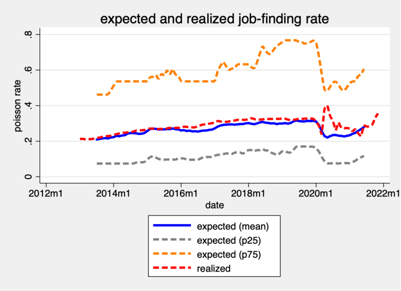


- Fit a truncated log-normal dist over the cross-section of PRs

Heterogeneity in UE risk

Perceived UE risks and realization





- realizations are computed from CPS panel data of workers following Fujita and Ramey, 2009

Individual PRs explain **their own** spending decisions

$$E_{i,t}(\Delta c_{i,t+1}) = u_0 + u_1 E_{i,t}(\Delta w_{i,t}) + u_2 \text{Var}_{i,t}(\Delta w_{i,t+1}) + \xi_{i,t}$$

	(1)	(2)	(3)	(4)	(5)
expected wage growth	0.324***	0.306***	0.254***	0.243***	
	(0.0825)	(0.0828)	(0.0334)	(0.0334)	
perceived wage risk	6.127***	6.185***	2.096***	1.711***	
	(1.163)	(1.165)	(0.439)	(0.442)	
perceived UE risk next 4m					0.353***
					(0.0553)
R-squared	0.000939	0.00318	0.953	0.953	0.633
Sample Size	56046	56046	56046	56046	6269
Time FE	No	Yes	No	Yes	Yes
Individual FE	No	No	Yes	Yes	Yes

- Higher perceived risks $\rightarrow$ higher expected spending growth.

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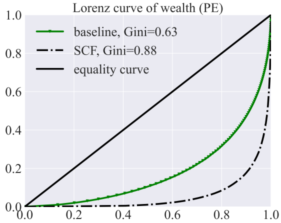
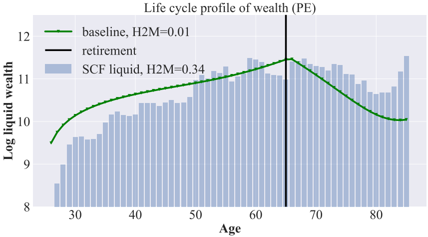
Model overview

- Overlapping generation
- Uninsured idiosyncratic income risks
  - Permanent+ transitory idiosyncratic wage shock
  - Persistent unemployment spells
- Partial/general equilibrium
- No aggregate risk a la Krusell and Smith, 1998
- A blend of Huggett, 1996 and C. D. Carroll, 1997
- Only one risk-free asset
- Calibrating income risks using survey versus estimates from panel
- Extension: subjective model
  - subjective PR $\neq$ objective income risks

StE distribution in the baseline model

- $\sigma_\psi = 0.15, \sigma_\theta = 0.15, U2U = 0.18, E2E = 0.96$ other parameters
- H2M: net liquid asset $<$ half-month income Kaplan, Moll, and Violante, 2018





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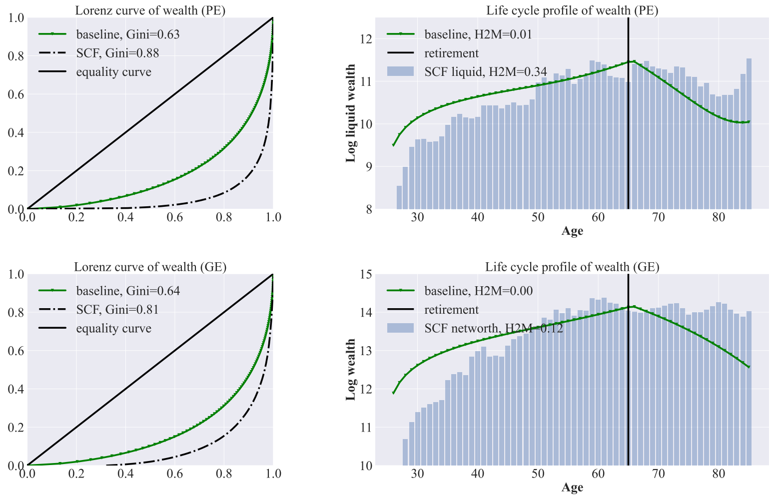
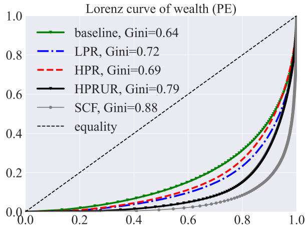
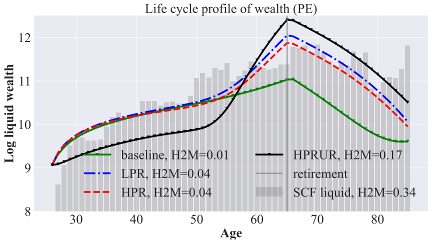


Model Comparisons





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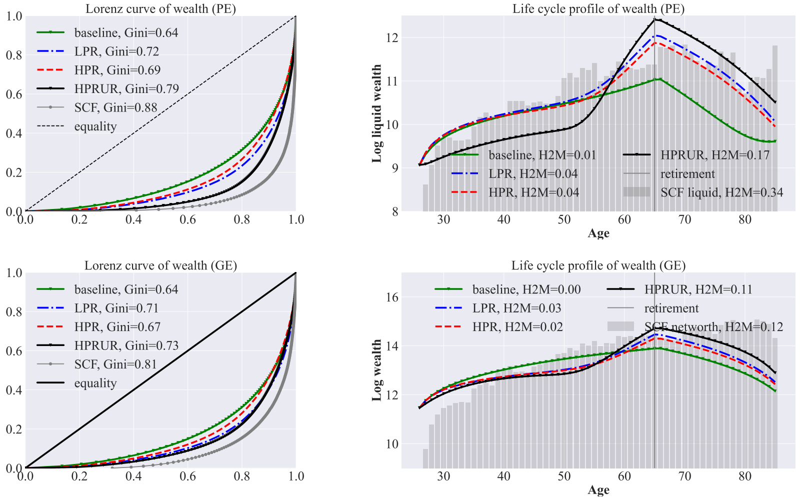


HPRUR: $\sigma_\psi = \sigma_\theta = [0.01, 0.02, 0.04]$, $U2U = [0, 0.02, 0.24]$, $E2E = [0.96, 0.99, 1.0]$

Model comparison

Model/Data	Gini	Bottom 0.9	Bottom 0.7	Bottom 0.5	Mean wealth/income ratio	H2M share
SCF (liquid)	0.88	0.18	0.04	0.01	0.67	0.34
baseline (PE)	0.64	0.47	0.22	0.10	1.17	0.01
LPR (PE)	0.72	0.40	0.15	0.06	1.06	0.04
HPR (PE)	0.69	0.45	0.17	0.07	1.03	0.04
HPRUR (PE)	0.79	0.33	0.08	0.03	0.70	0.17
SHPRUR (PE)	0.81	0.29	0.08	0.03	0.78	0.16
SCF (net worth)	0.81	0.29	0.09	0.02	6.72	0.12
baseline (GE)	0.64	0.47	0.22	0.10	2.17	0.00
LPR (GE)	0.71	0.41	0.15	0.07	1.20	0.03
HPR (GE)	0.67	0.46	0.18	0.08	1.23	0.02
HPRUR (GE)	0.73	0.41	0.14	0.06	1.12	0.11
SHPRUR (GE)	0.76	0.35	0.12	0.05	1.22	0.10

Extension: subjective PR

Key assumption:

- Ex-ante: saving decisions $\leftarrow$ subjective PRs
- Ex-post: realized income inequality $\leftarrow$ objective size of income risks

Two purposes:

- A robustness check: what if PRs are incorrect, e.g. over-confident
  - but we did find people behave according to their PRs
- A model breakdown into ex-ante “choice” and ex-post “shock” channels

Evolution of the distribution over state variables

- objective:

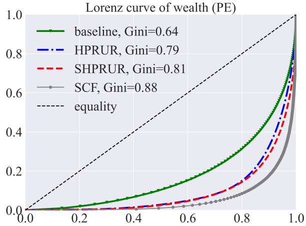
$$\psi_\tau(B) = \int_{x \in X} \underbrace{P(x, \tau - 1, B)}_{\text{transition funcs}} \mathrm{d}\psi_{\tau-1} \quad \text{for all} \quad B \in B(X)$$

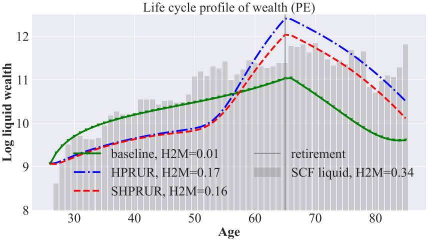
- subjective:

$$\tilde{\psi}_\tau(\tilde{B}) = \int_{\tilde{x} \in \tilde{X}} \tilde{P}(\tilde{x}, \tau - 1, \tilde{B}) \mathrm{d}\tilde{\psi}_{\tau-1} \quad \text{for all} \quad \tilde{B} \in \tilde{B}(X)$$

  - $\tilde{P}$ depends on both subjective and objective risks

Subjective (SHPRUR) v.s. Objective (HPRUR)





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- People's saving behaviors better explained by their perceptions
  ... than what economists assume to be their perceptions
- Survey data can inform incomplete-market macro models
  - Direct evidence for heterogeneity in perceptions that *matter*
  - Closer to agents' information set that truly affects their decisions
- More work needed on
  - heterogeneous beliefs in HM models
  - understanding risk perception formation
  - business cycle implications of perceived risks

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    - "heterogeneity or risk": Cunha, Heckman, and Navarro, 2005, Primiceri and Van Rens, 2009, Guvenen and Smith, 2014
    - "insurance or information": Pistaferri, 2001, Kaufmann and Pistaferri, 2009, Meghir and Pistaferri, 2011, Kaplan and Violante, 2010
- subjective/probabilistic survey of beliefs: Manski, 2004, Delavande, Giné, and McKenzie, 2011, Manski, 2018, Bertrand and Mullainathan, 2001, Armantier et al., 2017
- incomplete market macro: Bewley, 1976, Aiyagari, 1994, Huggett, 1996, Krusell and Smith, 1998, Heathcote, Storesletten, and Violante, 2009, C. Carroll et al., 2017, Krueger, Mitman, and Perri, 2016, Bayer et al., 2019
- consumption/saving under incomplete information/imperfect perception: Pischke, 1995, Wang, 2004, Rozsypal and Schlafmann, 2017, C. D. Carroll,

Benchmark model

$$\max \quad \mathbb{E}\left[\sum_{\tau=0}^{\tau=L-1}(1-D)^{\tau}\beta^{\tau}u(c_{i,\tau})\right]$$

$$\underbrace{a_{i,\tau}}_{\text{Savings}} = \underbrace{m_{i,\tau}}_{\text{Cash in hand}} - c_{i,\tau}$$

$$m_{i,\tau+1} = a_{i,\tau}R + (1 - \underbrace{\lambda}_{\text{Income tax}})(1 - \underbrace{\lambda_{SS}}_{\text{SS tax}})y_{i,\tau+1}$$

$$a_{i,\tau} \geq 0$$

- CRRA: $u(c) = \frac{c^{1-\rho}}{1-\rho}$
- Work age: $\tau = 1, 2, ..., T$; retirement : $\tau = T+1, ..., L$ (since entering job market)
- Survival probability: 1-D

Income process over the life-cycle

- income

$$y_{i,\tau} = n_{i,\tau} W$$

$$n_{i,\tau} = p_{i,\tau} \xi_{i,\tau}$$

- permanent component

$$p_{i,\tau} = G_\tau p_{i,\tau-1} \psi_{i,\tau}, \quad log(\psi_{i,\tau}) \sim N(-\sigma_\psi^2/2, \sigma_\psi^2) \quad \forall \tau \leq T$$

Income process over the life cycle

- income

$$y_{i,\tau} = n_{i,\tau} W$$
$$n_{i,\tau} = p_{i,\tau} \xi_{i,\tau}$$

- persistent/transitory component

$$\xi_{i,\tau} = \begin{cases} \theta_{i,\tau} \quad \text{if} \quad \nu_{i,\tau} = e \quad \& \quad \tau \leq T, \quad log(\theta_{i,\tau}) \sim N(-\frac{\sigma_\theta^2}{2}, \sigma_\theta^2) \\ \zeta \quad \text{if} \quad \nu_{i,\tau} = u \quad \& \quad \tau \leq T \\ \mathbb{S} \quad \text{if} \quad \tau > T \end{cases}$$

- transition probability between $\nu = u$ and $\nu = e$

$$\pi(\nu_{\tau+1} | \nu_\tau) = \begin{bmatrix} \mho & 1 - \mho \\ 1 - E & E \end{bmatrix}$$

Macroeconomic environment

- Technology

$$Y = ZK^{\alpha}N^{1-\alpha}$$

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- Government (balance budget)

$$\lambda\left[1 - \Pi^{\mho} + \zeta\Pi^{\mho}\right] = \zeta\Pi^{\mho}$$

$$\lambda_{SS}\sum_{\tau=1}^{T} G_{\tau}(1 - \Pi^{\mho}) = \mathbb{S}\sum_{\tau=T+1}^{L} G_{\tau}$$

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- Demographics
  - Stable age distribution $\{\mu_{\tau}\}_{\mu=1,2,..L}$

$$\mu_{\tau+1}=(1-D)\mu_{\tau},\quad \sum_{\tau=1}^{L}\mu_{\tau}=1$$

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Value function and transitions

- Value function

$$V_\tau(\underbrace{\nu_{i,\tau}, m_{i,\tau}, p_{i,\tau}}_{x_{i,\tau}}) = \max_{\{c_{i,\tau}, a_{i,\tau}\}} \quad u(c_{i,\tau})$$

$$+ (1 - D)\beta \mathbb{E}_\tau \left[V_{\tau+1}((\nu_{i,\tau}, m_{i,\tau+1}, p_{i,\tau+1})\right]$$

- Transitions

$$\psi_\tau(B) = \int_{x \in X} \underbrace{P(x, \tau - 1, B)}_{\text{transition funcs}} \mathrm{d}\psi_{\tau-1} \quad \text{for all} \quad B \in B(X)$$

  - $B(X)$: distribution measure on state space X
  - ψ_τ: distribution over state variables x for agents in age τ
  - ψ_1 depends on initial draws of income shocks

Stationary equilibrium (StE)

- Optimal consumption and saving policies given W, R, λ
- Distribution evolution consistent with optimal c and a policies and income risks
- The factor markets clear

$$\sum_{\tau} \mu_\tau \int_X a(x,\tau) \mathrm{d}\psi_\tau = K$$

$$\sum_{\tau=0}^{T-1} \mu_\tau \Pi_\tau^E = N$$

- Firm optimization under competitive factor markets.

$$W = Z(1-\alpha)(K/N)^\alpha$$

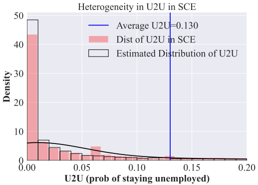
$$R = 1 + Z\alpha(K/N)^{\alpha-1} - \delta$$

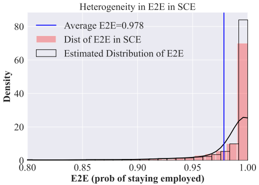
Calibration of the benchmark model

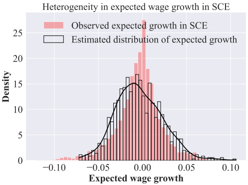
Block	Parameter name	Values	Source
risk	σ_ψ	0.15	Median estimate from the literature
risk	σ_θ	0.15	Median estimate from the literature
risk	$U2U$	0.18	Median estimate from the literature
risk	$E2E$	0.96	Median estimate from the literature
initial condition	$\sigma_\psi^{\text{init}}$	0.629	Estimated for age 25 in 2016 SCF
initial condition	bequest ratio	0	assumption
life cycle	n	0.005	U.S. census
life cycle	T	40	standard assumption
life cycle	L	60	standard assumption
life cycle	$1-D$	0.994	standard assumption
preference	ρ	2	standard calibration
preference	β	0.96/0.98	standard calibrations
policy	$\mathbb{S}$	0.65	U.S. average
policy	λ	N/A	endogenously determined
policy	λ_{SS}	N/A	endogenously determined
policy	μ	0.15	U.S. average
production	W	1	target values in steady state
production	K2Y ratio	3	target values in steady state
production	α	0.33	standard assumption
production	δ	0.025	standard assumption

Calibrating heterogeneous PRs

- Fit a truncated log-normal dist over the cross-section of PRs

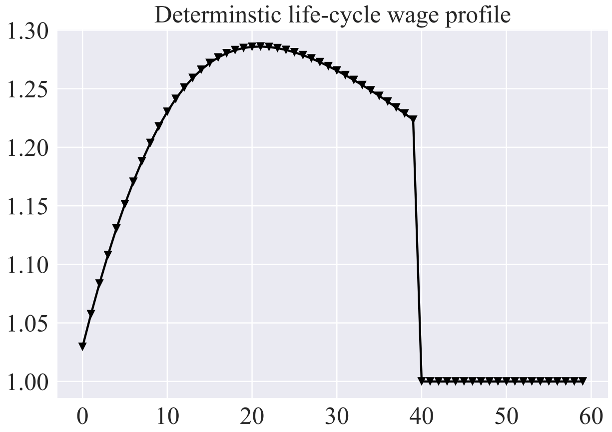






Back

Deterministic wage profile over life cycle



- Estimated from SIPP with a fourth-order age polynomial regression