Mathematical Preps for Element of Macroeconomics

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1 Percentage change, annualized rate, etc

- Percentage change
  - Many economic variables are expressed in percentage change terms
    - GDP growth rate
    - CPI Inflation
    - Population growth
  - The percent change of X from $t-1$ to t is computed as

$$\text{Percent change} = \frac{X_t - X_{t-1}}{X_{t-1}} \times 100\% \tag{1}$$

  - For instance, the GDP of an island is 100 in year 2001 and 105 in year 2002, then the GDP growth rate is $(105-100)/100 = 5\%$.
- Compunding growth
  - Sometimes we are interested in computing the compounded growth rates. For instance, the GDP grows by 2% the first year and -3% the second year, what is the growth rate over the two years?

$$\text{Growth rate} = (1+2\%)(1-3\%) - 1 = -1\% \tag{2}$$

  - You may want to use a calculator to compute the above. But a mathematical trick is sufficient. If $a\%$ and $b\%$ are small, i.e. single-digit percentage growth, we can approximate the compounding growth using the formula below.

$$(1+a\%)(1+b\%) \approx 1 + a\% + b\% \tag{3}$$

- Annualized growth rate

- Growth rates of something over the course of one year cannot be comparable to that over one month, or one day. So conventionally, economists like to express all growth rates for a fixed length of time, one year. To put it differently, economists want to **annualize** them.
- We can convert both growth rates over a period shorter or longer than one year. Below are two examples for both, respectively.
- Example 1. Assume that the housing price of the Baltimore grows by 0.5% each month, by how much would the housing price grow over a year?

$$\underbrace{(1+0.5\%)}_{\text{month 1}}\underbrace{(1+0.5\%)}_{\text{month 2}}\ldots\underbrace{(1+0.5\%)}_{\text{month 12}}-1=(1+0.5\%)^{12}-1 \tag{4}$$

  where 12 is the number of months a year has. Hence, use 4 if you are annualizing quarterly growth rates.
- Example 2. Assuming the GDP per capita in an island grows 4% from June 2018 to December 2020, what is the average annual growth rate during this period? Assuming the annual growth rate is g, then we have

$$(1+g)^{2.5}=(1+4\%)\rightarrow(1+g)=(1+4\%)^{\frac{1}{2.5}}\rightarrow g=(1+4\%)^{\frac{1}{2.5}}-1 \tag{5}$$

  where 2.5 is the number of years between the two points of the time.
- So the general formula for the annualized growth rate between t_1 to t_2

$$\text{Annualized growth}=(1+x)^{1/(t_2-t_1)}-1 \tag{6}$$

  where x is the growth rate between t_1 and t_2, and t_1 - t_2 is measured in years.

2 Slope

The slope of a straight line is computed as the following. (See Figure 1)

$$\text{Slope}=\frac{Y_2-Y_1}{X_2-X_1} \tag{7}$$

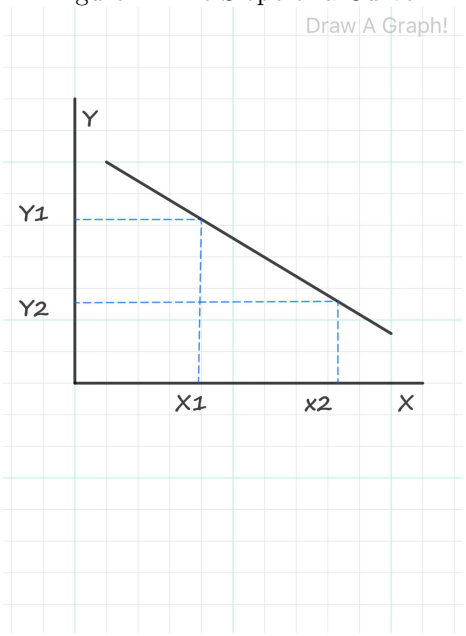
"Rise over run."

The slope can be negative, positive, or zero. A horizontal line has a slope of zero and a vertical one has a slope of infinity.

3 Movement along a curve v.s. a shift of a curve

A curve describes the relationship between X and Y, for instance, $Y=2X+b$ where b is a constant.

Figure 1: The Slope of a Curve



A movement along the curve means Y's value changes as X changes. See Figure 2.

A shift of the curve means for the same X, the b changes. See Figure 3.

4 Geometric sum

- A highly useful formula is the sum of a geometric sequence of numbers

$$a + ar + ar^2 + \ldots + ar^n = a\frac{1-r^n}{1-r} \tag{8}$$

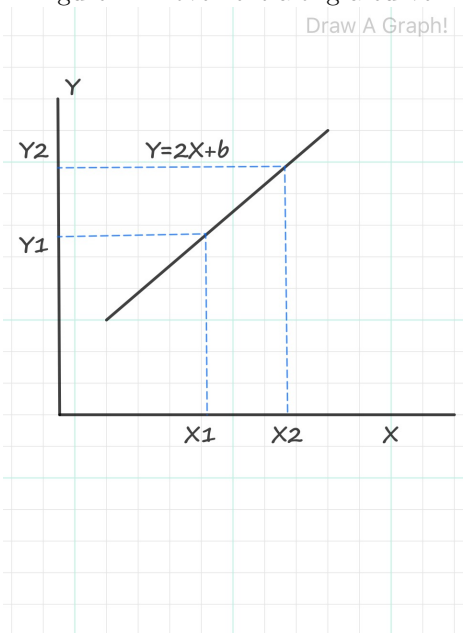
- If you extend the sequence foroever and if $-1 < r < 1$, then the sum is

$$a + ar + ar^2 + \ldots = a\frac{1}{1-r} \tag{9}$$

- This formula is important in macroeconomics. Imagine everyone in the economy gets \$1 and spends 0.2 fraction of it to buy something from another person. The person who receives the \$0.2 then spends a 0.2 fraction of that \$0.2 to buy something from a third person. If such a process continues forever, what would be the total spending resulting from that \$1 in the beginning? It is essentially

$$0.2 + 0.2^2 + 0.2^3 + \ldots = \frac{1}{1-0.2} = 1.25 \tag{10}$$

Figure 2: Movement along a curve



- Why does \$1 person leads to \$1.25 spending? This is called the **multiplier effect**. You will learn this later in this class. 0.2 here is called "marginal propensity to consume" (MPC) and 1.25 is called "spending multiplier".

5 Dealing with ratios

For a ratio X defined as following

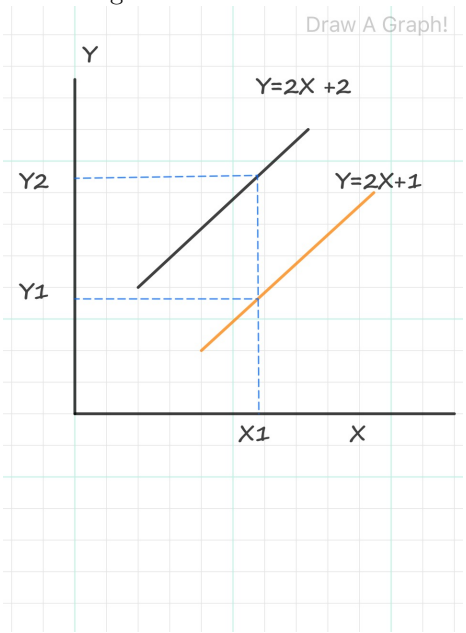
$$X = \frac{N}{D}$$

- If N grows by $n\%$ and D grows by $d\%$ a year, the growth rate of X, x is

$$x = n\% - d\%$$

  - Example 1. What is the growth rate of labor force participation rate(LFPR) if labor force increases by 1% and total working age population decreases by 0.5%?
    Answer: $1\% - (-0.5\%) = 1.5\%$.

Figure 3: Shift of a curve



Comparative Advantage and Absolute Advantage

Tao Wang

January 31, 2022

1. **Production possibility frontier (PPF)**
   - Suppose a country A can produce two kinds of products, i.e. wine and cheese. Since the resources in the country are limited, it can either use all resources to produce a certain amount of wine, a certain amount of cheese, or some amount of both. Production possibility frontier (PPF) allows us to graphically capture this notion of "how much you can produce" (Figure 1).
   - The line is called "frontier" because it is the maximum production that can be achieved when all resources are used (Figure 1).
   - The whole triangle area within the line is all the possibility since the country does not necessarily use up all the resources. It is "possible" to produce that amount (Figure 1).

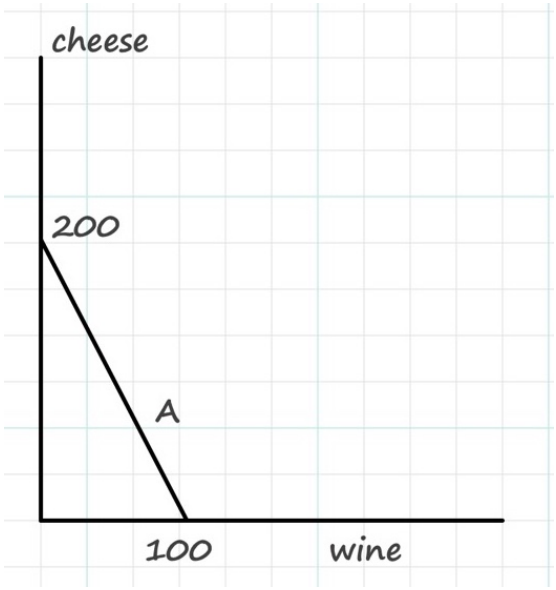


Figure 1: PPF of Country A

2. **Opportunity cost**

   - The absolute value of the slope of the PPF is the opportunity cost of **producing the good on the horizontal axis** measured by the units of the good on the vertical axis.
   - Opportunity cost of producing something is defined as the **maximum** production of another good that has to be given up.
   - More generally, *the opportunity cost of doing A (giving up B)* takes the following form. Be careful with the positions of numerators and denominators.

   $$\text{Opportunity cost of producing A } = |\frac{\Delta B}{\Delta A}|$$

   - So far, we are only talking about the opportunity cost of producing one good by one country. Do not confuse them with any comparisons between two countries in terms of absolute and comparative advantages.

3. **Absolute advantage** means the following

   - Country A can produce one good more productively than country B. If production possibility frontiers of A and B pass $(0, y_A)$ and $(0, y_B)$, respectively, we say if $y_A > y_B$, A has the *absolute advantage* over B in producing cheese.
   - More accurately speaking, country A uses less resource to produce one unit of cheese than B does. Let us assume that country A needs L_A^c labors for one unit of cheese and country B needs L_B^c labors. Then A has an absolute advantage over B in producing cheese means $L_A^c < L_B^c$.
   - Given the same amounts of labors in A and B, country A can produce more cheese than B does. Assume total labors in A and B are both L, then it means $L/L_A^c > L/L_B^c$.

4. **Comparative advantage** is a different concept. It speaks to not for one good, but two goods. This is why we call it "comparative".

   Country A has a comparative advantage over B in producing cheese means the following:

   - The relative productivity of producing cheese and wine is higher than country A than in country B.
   - It is equivalent to that the opportunity cost of producing cheese is lower in country A than in country B. (You are better at doing things with low opportunity cost. A life lesson...)
   - The slope of production frontier line with the vertical axis being cheese is steeper for country A than in country B.

- A simple version (enough for our class) : it means $y_A/x_A > y_B/x_B$, namely, the relative amount of cheese produced relative to wine is higher in A than in B.
- A more accurate version: relative productivity of cheese to wine is higher in A than in B. i.e. $L_A^c/L_A^w < L_B^c/L_B^w$.

5. **Difference and Relationship**

   The key difference between the two concepts:

   AA is for one good; CA is for at least two goods.

6. **Graphical illustration**

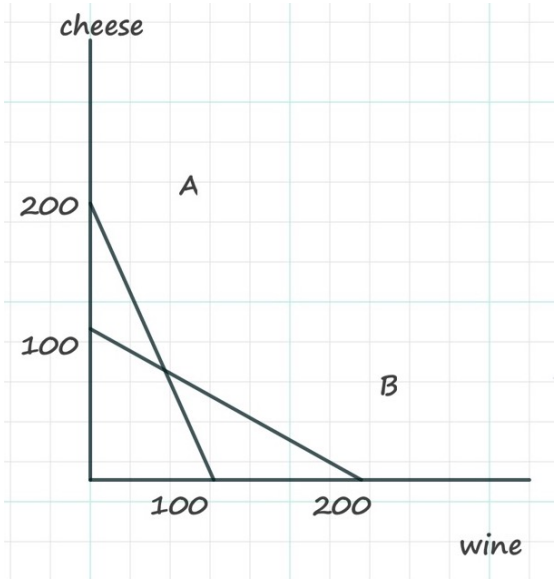


Figure 2: Production possibility frontier

- Intercept stands for production from specialization.
- Negative slope is the opportunity cost (relative productivity).

Now let us think about how to combine the production possibility frontiers of the two countries.

To fix idea, we start from one country, say country A. We know country A can at most produce either 100 wine or 200 cheese.

Now since we know that it is country B who has the comparative advantage of producing wine over cheese relative to country A, we are wondering what would happen to total production if country B only focuses on producing the good on which it has a comparative advantage, wine. That is 200 units

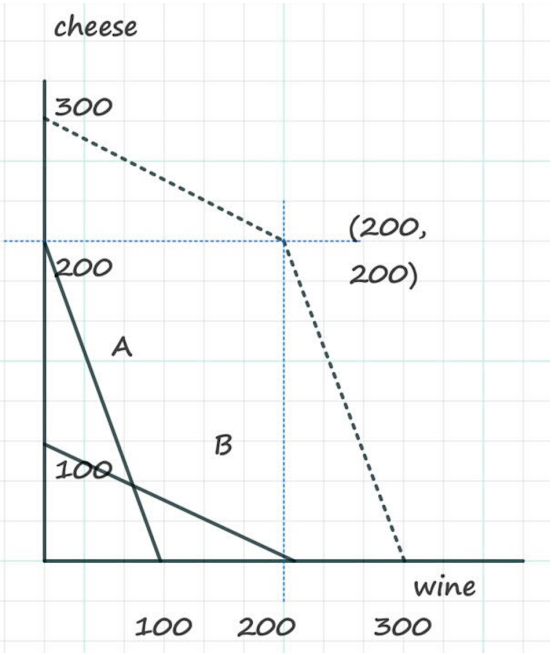


Figure 3: Combined PPF and gains from trade

of wine. Then what we do is basically move the PPF of A horizontally to the new dashed line in parallel to previous PPF of A in Figure 3. For any point in the PPF of A, add 200 more units of wine.

Following the same logic, we can move up the PPF of country B vertically by 200 since country A has the comparative advantage of producing cheese.

The two new dashed lines intersect at the point where either country specilizes in the good on which it has comparative advantage. This point together with the two intercepts make a combined production possibility frontier of the two countries.

A special case of the combined PPF remins a line without a kink. This is the case when the opportunity cost of producing either good by two countries are the same, or the two lines have the same slopes.

A short-cut to combining two PPFs is the following.

(a) Determine the comparative advantages of the two countries.

(b) The intercepts are the total production of each good by two countries when both only produce that good.

(c) The kinked point of combined PPF can be determined by letting the two countries specialize in producing the good on which they have a comparative advantage in producing, respectively.

National Income Accounting, Nominal/Real, Price Index and Inflation Indicators

Tao Wang

February 7, 2022

1 National Accounting

- Definition of GDP: the total value of final goods and service produced within the border of the country over a period of time.
  - Only final product, not intermediate. Avoids double counting.
  - Does not deduct depreciation. This is why it is called gross.
  - Includes inventory. To put it another way, once it is produced as a final product, it counts as GDP, even if it does not get sold.
  - Its definition does not preclude including informal economy (underground economy), and illicit activities. But in reality, it has not been measured well.
  - Does not include non-market activities such as housework and childcare (But we probably should include it.).
  - Does not take into account a wide range of factors that affect people's welfare, i.e. environment pollution, subjective happiness, etc.
  - Says nothing about the income distribution.
  - Says nothing about the efficiency use of the resources. Dig a hole on the ground and fill it. That is part of the GDP even if it is a waste of resource.
  - Difference with GNP.
    - GDP includes production by foreign residents within the country, while GNP does not.
    - GDP excludes production by citizens outside of the country, while GNP counts it.
    - Example 1. The sale revenue of Ernest Hemingway's Old Man and Sea in the U.S. (written in Cuba) counts as U.S. GDP? (No) U.S. (Yes) GNP? Cuba GDP (Yes)?
  - It is a flow concept instead of a stock concept.

    - * Flow. change from a point of time to another. i.e. income this year, consumption this year, investment this year, etc.
    - * Stock: the level of the variables at any particular point of the time. The volume of the swimming pool, national wealth, capital stock, etc.
  - It is in monetary value, instead of in the units of the products. Therefore, the prices enter in the GDP.

- GDP, GNP, and Gross National Income

$$\text{GNP} - \text{Depreciation} = \text{Net GNP}$$

$$\text{GNP} - \text{Statistical Discrepancy} = \text{Gross National Income}$$

- Different approaches to counting GDP.
  - Expenditure approach.

$$\text{GDP} = \underbrace{\text{Consumption}}_{\text{Personal Consumption+Government Consumption}} + \text{Investment} + \text{Net Exports}$$

    - * Can be thought as counting the product by the "use" of the income.
    - * Government spending does not include transfers.
  - Income approach.

$$\text{GDP} = \text{Household Income} + \text{Firms' Income} + \text{Government Income}$$

    - * Basically, by source.
    - * Financial income is not treated as income if it is purely a transfer from losers to winners in the market.
  - Value-added approach.
    - * Directly look at the production sector. Since we only focus on final products, we need to deduct the intermediate goods to avoid double counting.

- An example.
  - An island with only one orange farm which hires one worker.
  - The farm produces $10 worth of oranges.
  - The worker in the farm earns $5.
  - Then a factory turn these oranges into orange juice.
  - The factory hires one worker.
  - Factory worker is paid by $10.

- The factory spends $10 on buying all the oranges from the farm.
- The factory produces juice worth of $100.
- The factory sells $80 worth of juice (to whom? to the two workers!) with $20 inventory.
- The sale revenue of the orange juices either go to the worker or going to the firm.
- **Value-added approach**. From seeds to orange, value-added is $10. From oranges to juice, $90. Total value-added is $100.
- **Income approach**. Worker salary is $10+$5 = $15. Income to the farm is $10 - $5 = 5. Income to the factory is $ 100 -$10 -$10= $80. Total income is $100.
- **Expenditure approach**. Workers consumption is $80. Investment is $20 worth of inventory. No government. So in total is $100.

2 Nominal and Real

- Why do we make the distinction?
  - Comparing the monetary value in different periods of time requires that the units remain the same. You cannot simply say $100 in 2019 and $100 in 1979 have the same value.
  - Wage needs to be measured by how much of the goods it can buy.
  - The interests earned from deposit accounts need to be adjusted to reflect the change in its purchasing power. For instance, you earn a 2% deposit interest next year, but the price becomes more expensive by 3%. Then you lost!
  - GDP is the monetary value of the total product. Its change comes from changes in quantity and price. We only care about the first, i.e. how much more things we produce?
- How we convert nominal to real.
  - Use GDP deflator.
  - Use the price of the base year to compute the GDP of different years.

3 Price Index and Inflation

- Price index is a weighted average of a basket of goods.

$$\text{PI} = p_1 w_1 + p_2 w_2 + .. + p_n w_n$$
$$w_1 + w_2 + ...w_n = 1$$

  - Weights.

    - Weights are fixed so that the price levels at different points of time are comparable.
    - Once for a while, weights need to be adjusted by the statistical agency to reflect the underlying change in the composition of households consumption.
    - Weights may be country-specific.
        - Include price of meat in the bundle of a vegetarian country? Not a good idea.
  - Consumer's basket.
    - Food is included.
    - Housing price is not included.
    - Housing rent is included.
    - In reality, every household has a different bundle. But we find a representative bundle for a average household.
  - Survey methods.
    - Random sampling across regions and type of retailers.
    - The surveying staff records the prices.

- Inflation is the percentage change in the price index.

$$Inflation = \frac{PI_t - PI_{t-1}}{PI_{t-1}} \times 100\%$$

  - Without knowing weights, one cannot compute the inflation even if the price change of each type of good is known.
    - Average of the change $\neq$ the change of average unless the weights are equal across different goods.

- Annualized inflation.

- Different measures of inflation
  - CPI
  - Core CPI, CPI excluding
    - food
    - energy
  - PCE
    - CPI is based on a survey of what households are buying; the PCE is based on surveys of what businesses are selling.
    - The CPI only covers out-of-pocket expenditures on goods and services purchased. It excludes other expenditures that are not paid for directly, i.e. medical spending paid by medical insurance.

        * CPI fixes the weights, while PCE accounts for substitution effect.

- Extended discussion.
    - Long-term change in good prices (See Figure 1).
    - Online shopping and price index.
        * Online price be included? How much weight?
        * Does it change the method of collecting price data?
        * Does it change the patterns of price?
            · Less dispersion across regions.
            · More frequent adjustment.

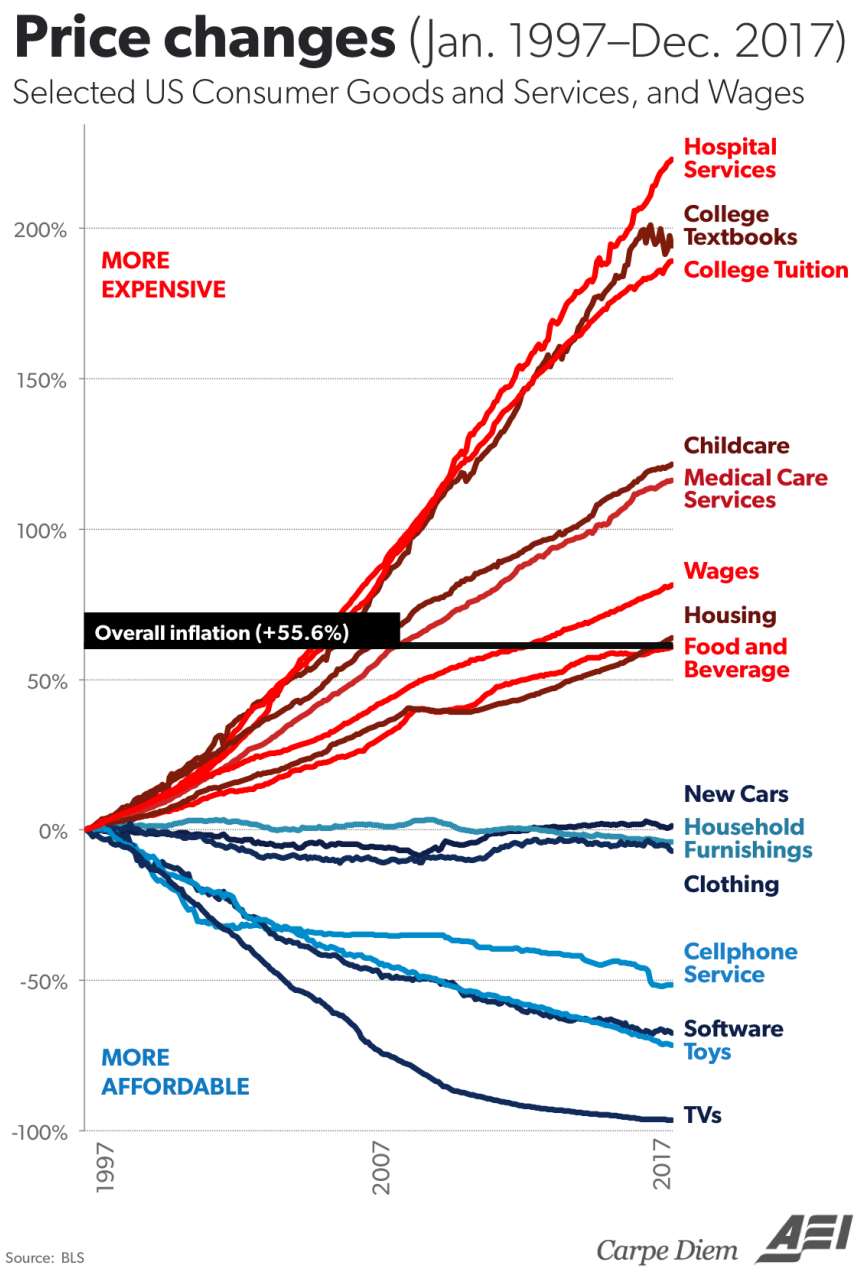


Figure 1: Chart of the century

Indicators of the Labor Market

Tao Wang

February 21, 2022

1 Unemployment rate and other concepts

1. Basics.

   Verbally speaking, the unemployment rate is the share of the labor force who are not employed but are available and willing to work. The numerator is the unemployed labor force. The denominator is the total labor force.

$$\text{Unemployment rate} = \frac{\text{The number of unemployed}}{\text{Labor force}} \times 100\%$$

- **Unemployed**
  - available to work
    - including absence for temporary illness
  - actively looking for a job over the past 4 weeks(1 month)
    - or in temporary laid-off while expecting to return the job
    - if not, not in labor force.
  - but does not have a job.
- **Labor force**
  - Who are out of labor force? (See the figure 1 below)
    - not available for working, i.e. young people, students in school.
    - the disabled
    - discouraged workers, those able to work but withdraw from work and searching for jobs, i.e. a healthy 25-year-old who choose to stay at home all day playing video games.
    - A laid-off worker who gave up the efforts to find for a job.
    - A young mom who quits her job and stays at home taking care of her baby.
  - Only **civilian** and **non-instituaional** labor force, excludes
    - workers in active duty in armed forces.

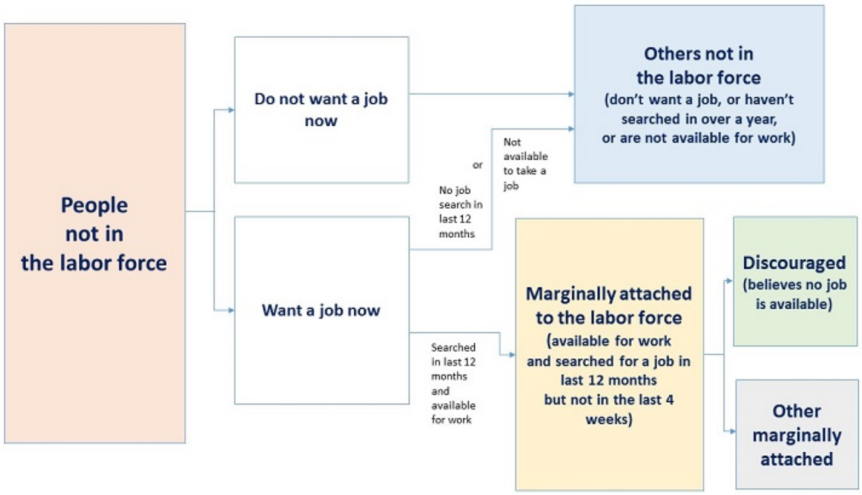


Figure 1: Out of the Labor Force

· non-military job in military is considerred as civilian

∗ prison workers.

– Civilian non-institutional population ⊃ labor force ⊃ the unemployed. (See Figure 1.)

- Type of unemployment
  - **Frictional UE**: someone who is in the search for a suited position after quitting her previous job.
  - **Structural UE**: the workers who were replaced by robots in Amazon warehouses.
  - **Cyclical UE**: the weak consumer confidence in the current quarter leads retailers to fire 20% of their cashiers. But they may hire some back when the economy picks up in the future.
- How do we get these numbers?
  - Household surveys. Ask households.
  - Payroll surveys. Ask firms.

2. Labor force participation rate (LFPR)

$$\text{LFPR} = \frac{\text{Labor force}}{\text{Working-age population}}$$

- Labor force defined as above.
- Working-age population is civilian and non-institutional.

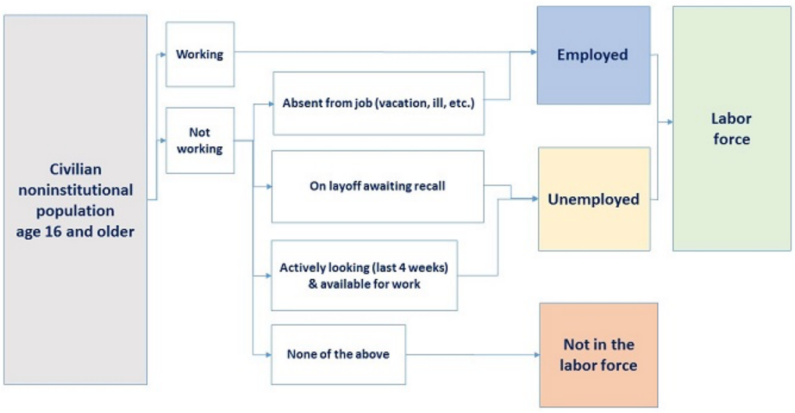


Figure 2: The Breakdown of Civilian noninstitutional population

- The working age in the U.S. is 16 or older.
- Either employed or unemployed, by definition.

3. Alternative measures of unemployment rates

   - U-3 is the official measure defined above.
   - U-6 is a broader measure.

     U-6 = U-3+the marginally attached to labor force+part-time workers for economic reasons

     - marginal attached workers: who have given up looking for jobs.
     - part-time workers for economic reasons: working less than 35 hours per week who want to work full time.

   - Example 1: Uber drivers.
     - Depending on how many hours worked.
   - Example 2: a video blogger who works at home.
     - The same as above.
   - Other measures (See Figure 3)
     - U-1, persons unemployed 15 weeks or longer, as a percent of the civilian labor force;
     - U-2, job losers and persons who completed temporary jobs, as a percent of the civilian labor force;
     - U-4, total unemployed plus discouraged workers, as a percent of the civilian labor force plus discouraged workers;

Table A-15. Alternative measures of labor underutilization

HOUSEHOLD DATA
Table A-15. Alternative measures of labor underutilization
[Percent]

Measure	Not seasonally adjusted			Seasonally adjusted					
	Aug. 2018	July 2019	Aug. 2019	Aug. 2018	Apr. 2019	May 2019	June 2019	July 2019	Aug. 2019
U-1 Persons unemployed 15 weeks or longer, as a percent of the civilian labor force	1.3	1.2	1.2	1.4	1.3	1.3	1.3	1.3	1.3
U-2 Job losers and persons who completed temporary jobs, as a percent of the civilian labor force	1.8	1.8	1.8	1.8	1.6	1.6	1.7	1.7	1.8
U-3 Total unemployed, as a percent of the civilian labor force (official unemployment rate)	3.9	4.0	3.8	3.8	3.6	3.6	3.7	3.7	3.7
U-4 Total unemployed plus discouraged workers, as a percent of the civilian labor force plus discouraged workers	4.2	4.2	4.1	4.1	3.9	3.8	3.9	3.9	4.0
U-5 Total unemployed, plus discouraged workers, plus all other persons marginally attached to the labor force, as a percent of the civilian labor force plus all persons marginally attached to the labor force	4.8	4.8	4.7	4.7	4.4	4.4	4.6	4.6	4.6
U-6 Total unemployed, plus all persons marginally attached to the labor force, plus total employed part time for economic reasons, as a percent of the civilian labor force plus all persons marginally attached to the labor force	7.4	7.3	7.3	7.4	7.3	7.1	7.2	7.0	7.2

NOTE: Persons marginally attached to the labor force are those who currently are neither working nor looking for work but indicate that they want and are available for a job and have looked for work sometime in the past 12 months. Discouraged workers, a subset of the marginally attached, have given a job-market related reason for not currently looking for work. Persons employed part time for economic reasons are those who want and are available for full-time work but have had to settle for a part-time schedule. Updated population controls are introduced annually with the release of January data.

Figure 3: Alternative Measures of Unemployment Rate

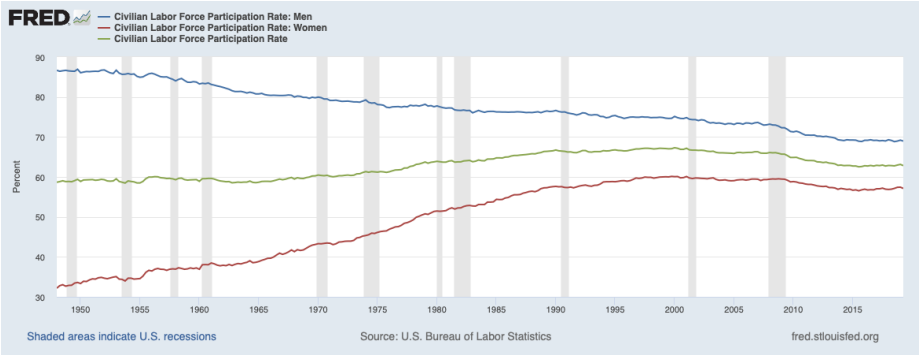


Figure 4: The Trend of Labor Force Participation of the U.S. Poulation

  - U-5, total unemployed, plus discouraged workers, plus all other marginally attached workers, as a percent of the civilian labor force plus all marginally attached workers;

4. Some examples.

   - Example 1. UE increased from 2.3% in 2010 to 5% in 2012. What drove the change?
     - The numerator ↑: more people found jobs given the same size of the labor force.
     - the denominator ↓: the number of people in work did not change, while the size of the labor force decreased.
   - Example 2.
     - There was a rise in LFPR of the total population between 1950-2000 while the LFPR of men decreased in the same period. What could you infer about the trend of LFPR of women? (See Figure 4.)

Reference

1. Glossary and Definition by BLS: https://www.bls.gov/cps/definitions.htm#laborforce

Economic Growth

Tao Wang

March 4, 2022

1 Decomposition of Growth Rate

- Long-run sustainable growth rate (LTSG)
- Why "long-run"?
  - We ignore the short-run business cycle fluctuations, i.e. economy could outperform / underperform from year to year simply because demand is higher or lower due to short-run reasons.
  - We typically take an average of growth rate over a long period of time to get its long-run growth rate.
  - For instance, the average growth rate of U.S. real GDP between 1950-2018 is 3.1%.
- What contributes to this growth in Y? The real GDP growth of an economy can be decomposed into two components

$$LTSG = \Delta LP + \Delta LF \tag{1}$$

  - Growth rate of labor force, i.e. more people working brings more output.
    - Note sometimes we measure LF by total number of hours worked instead of the number of workers. The key is that it measures how much labor input is used in the production.
    - Population growth, i.e. especially the population growth in prime-age workers.
    - A higher labor force participation rate for a given size of working age population, i.e. women's LFPR increased over the past decades.
    - Immigrants flows.
  - Growth rate of labor productivity, i.e. higher efficiency of using the same amount of labor input brings about higher output.
    - Technological progress/innovations.
    - More educated workers.

    - Better economic institutions that facilitate social cooperation.

- Some math
  - When A growth s at a percent and B grows at b percent a year, the annual grow rate of $A \times B$ is approximately $a + b$ percent a year.
  - It is also equivalent to the annualized growth rate of $A \times B$ is $(1 + a)(1 + b) - 1 = ab + a + b \approx a + b$.
  - In the example above, A is LP and B is LF.
- The formula above could be also used to "infer" the growth rate in productivity (which is not directly observable) using the growth rate of Y and LF, both of which are observed.

2 Solow Growth Model

Solow economic growth model more formally analyzes the drivers of economic growth described by the decomposition in the previous section.

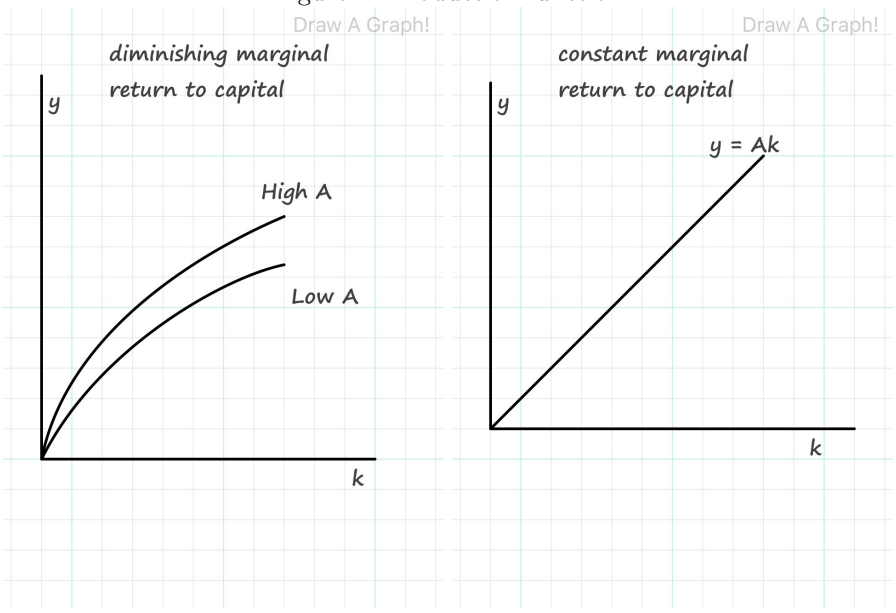
2.1 Production function

- A production function describes how a certain amount of factors of production are used as inputs and converted into a certain amount of outputs, and how different inputs are combined, exactly.
- In its most simplistic version: labor is the only input.
- More realistically, labor and capital as both inputs.
- The most common production function in macroeconomics, i.e. Cobb-Douglass production function takes the following form.

$$Y = AK^{\alpha}L^{1-\alpha}$$

  - A: total-factor productivity. The greater A is, the more is produced with the same amount of inputs, thus more productive.
  - K: the capital stock. (It is stock, not flow.)
  - L: labor.
  - α : the contribution of the input to the production. The greater α is, the more capital is required for the production. Another way of saying it, more capital-intensive.
  - K is to the power of α and L is to the power of $1 - \alpha$, the two summing up to one has to do with the constant return of scale (CRS) property. If increasing inputs by n times, one produces n times of the output. For instance, doubling the capital and labor exactly doubles the production.

Figure 1: Production function



$$A(2K)^{\alpha}(2L)^{1-\alpha} = 2AK^{\alpha}L^{1-\alpha}$$

- Economists conventionally analyze economic growth in terms of output per worker. This is because the size of the labor force is in starkly different magnitudes today compared to, say, 200 years ago. The aggregate production is not as informative as production per capita. $y = Y/L$, i.e. output per worker is total output divided by total number of workers or number of hours worked. $k = K/L$ capital per worker is equal to total capital divided by total number of workers or total number hours worked.
- Thus we divide the equation above by L on both sides of the equation.

$$\frac{Y}{L} = A(\frac{K}{L})^{\alpha}$$

$$\rightarrow y = \underbrace{A}_{\text{productivity}} \quad \underbrace{k^{\alpha}}_{\text{capital per worker}}$$

  where y is the production per capita and k is the capital per capita. We can plot the production function in the graph. (See Figure 2.1)
- Depending on if $\alpha < 1$ or $\alpha = 1$, the production function in per capita is decreasing return of scale or constant return of scale.

- A few observations we can make from the graph above.
  - α above is typically between 0 and 1. This corresponds to the **Diminishing marginal return from capital**, i.e. each additional unit of increase in capital per worker brings less and less additional output per worker. [1]
  - It means the marginal product of production with respect to capital per capita decreases with k;
  - Graphically, the slope of the production function is smaller with a higher k, or the curve become flatter and flatter as k increases.
  - Bigger A shifts the curve up. Higher productivity allows one to produce more with the same capital per capita.
  - If $\alpha = 1$, the production function curve becomes linear. (But not necessarily 45-degree line, which is the case only when $A = 1$.)

2.2 Sources of economic growth

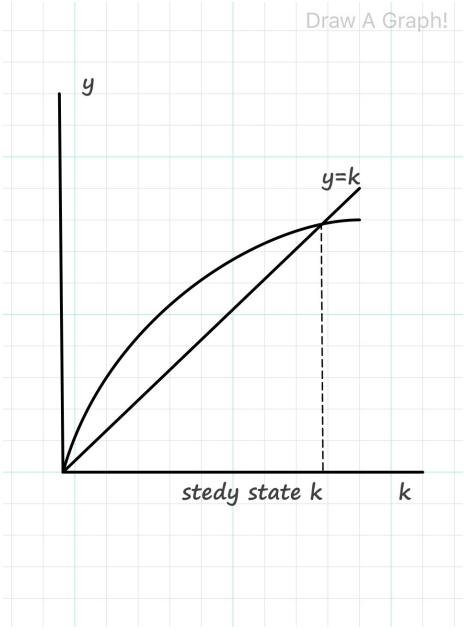
Using the production function $y = Ak^{\alpha}$, Solow model states that there are two ways of getting a higher output, thus driving the economic growth of a country.

- Higher productivity. $A \uparrow$. Factors that increase A: better education of labors, better institutions, better management, etc.
- Capital **deepening**. $k \uparrow$. Giving each worker more capital helps economic growth.
- Since **capital deepening** brings less and less increase in output, it cannot drive economic growth forever. Once it recedes, it is the innovation, the increase in A , that drives sustainable economic growth.
- **Solow residual**: if we first try to explain the observed economic growth per worker Δy by the observed change in capital per worker, Δk, we have something "left", which can be only due to chagne in A. This is why we call it "residual".
- There is a natural limit to growing by deepening physical capital. Why?
  - Decreasing marginal return, i.e. the return to capital or the marginal product of capital (MPK) decreases with k.
  - Therefore, an increase in the capital can only bring about less and less economic growth.
- Steady-state capital.

[1] For the discussion about economic growth, economists like to assume α is constant. Actually, it was a fair assumption for a long period of time. For instance, estimated from U.S. data, $\alpha_{U.S.} \approx 0.33$. But in the recent decade, there is mounting evidence that the capital share increases over time. This implies the share of income going to labors' decline. (Not good to income equality.)

- Until one point in which the new accumulation of the capital from growth in the previous year is only enough to keep you at the same level of the capital.
- This is called **steady state** capital.
- Because the capital you use for production jn next period comes from the saving of the previous year, which itself is proportional to your total production . If higher capital cannot increase the size of the saving fast enough, the capital accumulation stops at some point.
- Graphically, it is when the production function intersects with a 45-degree line of $y = k$. Once reaching that point, there is no way to attain growth from increasing capital. (See Figure 2)

Figure 2: Steady state capital in Solow growth model



3 Endogenous Growth

- Where is productivity growth from?
  - We see human society has grown for about hundreds and even a thousand years, it has to be due to the first channel, i.e. the productivity itself grows over time without reaching some natural limit yet.
  - Specifically, assuming productivity A is not a constant, but it grows at $a\%$ a year, then given the same capital k, the growth of the output per worker is

$$\%Y = \%A = a\%$$

  - But we remain silent about why A could grow at $a\%$.
  - We need to have a theory of why productivity itself, unlike the physical capital such as machines, land, factories, can grow without being constrained by the natural limit. To put it simply, why is the technology itself not decreasing return but increasing return?
  - It turns out that the magic lies in the distinctive feature of the factor that drives technological growth: the knowledge.

- What is special about knowledge?
  - Non-rival: one using the knowledge does not naturally preclude others from using it.
    - Do not confuse it with patent law protecting intellectual property.
  - Positive externality.

- As a result, a few important patterns arise.
  - As a byproduct of the production process, knowledge and know-how is generated that can be used for production in the future. This is as if the knowledge is accumulated as stock.
  - The bigger the stock of the knowledge, the more likely the new ideas are produced. This is the idea of increasing the return of scale (IRS).
  - Even though each firm seeks its own goal of profits-maximizing, their production and investment bring about positive externality to the society by contributing to the stock of the knowledge.

Okun's Law

Tao Wang

April 25, 2022

1 Okun's Law

- What is the use of Okun's law? You, an economist, or a policymaker, seeing the unemployment rate being 3% for the time being, remain clueless about where the economy is on its track. To put it another way, is 3% too good, or still not as good as the economy's better potential? You need a benchmark of the economy so that you can determine where you are really. As a rule of thumb, Okun's law describes such a handy relationship between change in unemployment from one time to another and the corresponding output growth for the same period.

  $$\%\Delta Y = LRSG - \text{Okun's Coefficient} * \Delta U$$

  where $\%\Delta Y$ is the change in output and ΔU is the change in the unemployment rate.

- $\%\Delta Y$ is the percentage growth rate of the output from one point of time to another.

- ΔU is in percent. i.e. $3\% - 2\% = 1\%$, in the same unit as the unemployment rate.

- LRSG, the long-run sustainable growth rate is obtained from the following equation.

  $LRSG =$ Growth rate of labor productivity + Growth rate of labor force

  .

  How do we get this? Assuming a production function that takes only labor as inputs in the following form.

  $$Y = AL$$

  The growth rate of Y is equal to the sum of the growth rate of A and L. Economic growth comes from either making the production more productive, i.e. each worker produces more, or bringing more people to work.

- Okun's law coefficient
  - If 1% decrease in UE, leads to exactly the employed population to grow by 1%, then we should expect the output to grow by 1%. Why? Output growth is equal to growth of labor given the same productivity. Therefore, the Okun's coefficient should be exactly 1.
  - But in economic expansions, the coeffiient is above 1. Because, numerically speaking, 1% decrease in UE brings about more than 1% increase of output. Why? One reason might be the employed population increases by more than 1%. This is due to the labor force, the denominator in the UE formula, increases as well. So an 1% decrease in UE means more than 1% of increase in employed population.
  - By the same token, the coefficient may be less than 1 in economic recessions.

- A few important things to be aware of regarding the application of Okun's Law.
  - It can be between any two points of the time. But it needs to be consistent on both sides. For instance, if ΔY is between 2017 and 2019, a two-year growth rate, then the change of unemployment rate ΔU needs to be also the difference of unemployment rates between 2017 and 2019.
  - More importantly, LRSR needs to be adjusted for the corresponding length of time horizon. For instance, in the example above, the two points of time are 2 years apart, therefore, you need to plug a two-year compounding rate of LRSG in Okun's law equation. Specifically, assuming LRSG is the annual growth rate, you need to use $(1 + LRSG)^2 - 1$ in the Okun's law equation.
  - Economists implicitly assume the coefficient is constant and it has been estimated from the historical data. But there are ongoing debates among researchers whether the coefficient is as stable as before.
  - LRSR can be seen as the a special case of Okun's law. Plug $\Delta U = 0$ in the equation of Okun's law, we have

    $$\%\Delta Y = LRSG$$

    This basically means that when the unemployment rate does not change anymore, in its **natural** level, then the economy exactly attains its long-run sustainable growth rate.

Aggregate Expenditure Model

Tao Wang

March 28, 2022

1 Aggregate Expenditure Model

1.1 What is aggregate expenditure

- Essentially, it is how much an economy as a whole plans to spend, or what macroeconomists call **aggregate demand**.
- What are the different components of the aggregate demand?
  - Consumption by households, of course.
  - Investment, i.e. firm needs to buy equipment to produce things. Investment also includes inventory, things that don't get sold and goes to their warehouses.
  - Government needs to buy things, i.e. military base, infrastructure, etc.
  - In an open economy, net exports, exports - imports, which you sell to other countries minus what you buy from countries.
  - This gives the following formula.

$$AE = C + I + G + NX$$

- How is C determined?

  There are two components. One component is not dependent upon the income. The other component increases with the income.

  - $\bar{C}$: autonomous consumption.
    - Necessities for basic living.
    - Consumption driven by sentiment.
  - Marginal Propensity to Consume $(MPC) \times Y$
    - how much you want to spend depends on how much you can make
    - Y: income

    - MPC governs the degree to which each additional unit of income turns into consumption.
        - **Marginal**: the fraction to be spent out of one **additional** dollar of income.
        - Very important to understand macroeconomics.
        - Factors that affect MPC
            1. Interest rate: higher interest rate makes it more worthwhile to put money in the bank, more savings, hence, less consumption, hence, lower MPCs.
            2. Wealth: wealthy households have low MPCs, i.e. think about giving a $100 check to Bill Gates and what he would do with it?
            3. Expected future income, better future income expectations lead to higher MPCs, i.e. no need to save as much as you would need in the low-income expectations scenario.
  - G is determined by policy makers. We will learn later that government could use G as a tool to change the equilibrium output.

- How is I determined?
  - More accurately, we should think I as **planned investment**.
    - Investment decisions are inherently forward looking.
    - Firms make plans based on their expectations of how much they can sell and where the future economy is going.
    - How much the firm invests depends on what how much they expect them to be able to sell.
  - But note that firms may turn out be "wrong", i.e. building too much inventory but fail to sell all of them. The **unplanned** investment needs to go up or down depending on how the economy realized.

1.2 How is the the equilibrium determined?

- The equilibrium of the economy is when the aggregate expenditure is equal to the total output or income Y.

$$Y = AE \tag{1}$$

- A more intuitive way of thinking of this is that what you plan to spend as an economy (AE) is equal to the amount of income the economy ultimately generates.

- Mathematically, we need to solve the equilibrium income Y^* by finding the Y that satisfies the following equation

$$\bar{C} + MPC \times Y + I + G + NX = Y \tag{2}$$

which gives

$$Y^* = \frac{\bar{C} + I + G + NX}{1 - MPC} \tag{3}$$

- Graphically, we can draw a 45-degree line, corresponding to $AE = Y$, in addition to the AE line, the intersection of the two lines is the equilibrium output/income of the economy.

1.3 The relationship between the AE and GDP

Think of the real GDP as the *realized* output/income of the economy, which could be, in theory, different from the Y^* we solved above. Why? Because that Y^* is under the assumption that firms perfectly plan their investment, such that the *planned* investment (PI) is equal to *actual* investment(AI).

$$\text{PI} = \text{AI} \tag{4}$$

or equivalently, the unplanned investment (UPI) is equal to zero.

$$\text{UPI} = \text{AI} - \text{PI} = 0 \tag{5}$$

But it is often unlikely that what actually happens in the economy goes as planned. Therefore, the realized real GDP could be lower (left to Y^* in Figure 1) or higher than Y^* (right to Y^* in Figure 1). The former could happen, for instance, when aggregate demand turns out to be weak, and firms could not sell all their products and unexpectedly increase their inventory (thus investment). This means $UPI > 0$ or $AI > PI$. The latter happens when the aggregate demand turns out to be stronger than the firms predicted and the inventories go down faster than planned. Hence, $UPI < 0$, or $AI < PI$.

For any value of realized GDP, say GDP, we can solve the corresponding value of the unplanned investment (UPI) by plugging $RGDP$ in the following equation.

$$\bar{C} + MPC \times GDP + PI + UPI + G + NX = GDP \tag{6}$$

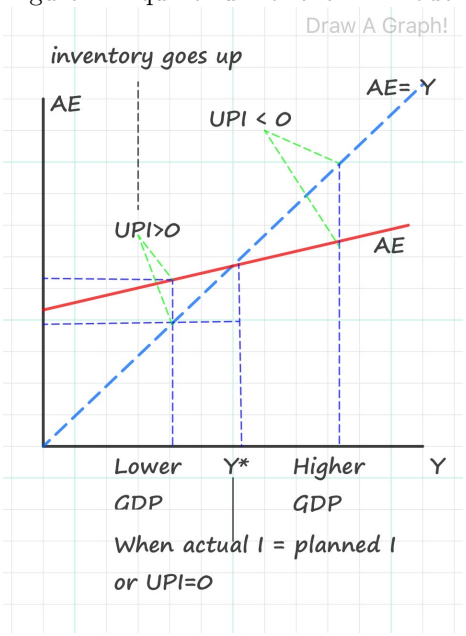
It turns out even easier to find the answer directly in the graph, as in Figure 1. The size of the UPI is the difference between AE line and 45-degree line, with the additional adjustment of the sign depending the value of GDP is left or right to the Y^*.

1.4 Assumptions behind the AE model

AE model is a model that explains why macroeconomy is determined by the demand (instead of supply) in the short-run. To put it more bluntly, "the more you (as the economy as a whole) want, the higher the output."

Obviously this shall not hold forever. A higher demand will make things more expensive, which would dampen demand after all. Also, ultimately the

Figure 1: Equilibrium of the AE model



resources are limited and there is a limit to how much the economy is able to produce regardless of the demand.

Therefore, two fundamental assumptions are critical for the AE model to make sense:

- There is "slack" in the economy. There are unused resources in the economy that can be used to produce more.
  - If all resources are used already, i.e. no slack, how much the economy can produce will not depend on the aggregate demand.
- No prices adjustment, or prices are fixed.

2 Multiplier effect

We want to understand the effect of change in G (similarly for I, $\bar{C}$) on the equilibrium Y^*. How much increase in the equilibrium output Y^* is induced by one \$1 more government spending in the economy.

- Intuitively, the \$1 increase in G, once spent, becomes the income of people in the economy. Then the people who get that \$1 would spend a MPC fraction of it, which becomes income of the people in the economy, again. Then a MPC fraction of that income will be spent, which is MPC^2. This

chain of the effect will continue "forever". The total effect can be summarized by an infinite sum of the following geometric sequence.

$$MPC + MPC^2 + MPC^3 \ldots = \frac{1}{1 - MPC} \tag{7}$$

- Notice here the multiplier $= 1$ when MPC=0. For any positive $MPC < 1$, the multiplier effect is greater than 1. That is why is called a multiplier.
- A higher MPC means a bigger multiplier effect. Therefore, when the MPC is high, the stimulating effect from an increase in the government spending is bigger.

Aggregate Supply and Demand

Tao Wang

March 18, 2022

1 How to Differentiate Supply and the Demand Shocks in Macroeconomy

- Aggregate supply

  Basically, any factors that change how much the firms want to produce for a *given* overall price level affects the supply side of the economy. (A shift of the SRAS curve or LRAS curve, depending on if the shock is transitory or permanent.)

  - Short-run AS
    - Labor cost
      - Promulgation of the minimum wage law. Labor cost ↑, thus $SRAS$ ←.
      - The bargaining power of labor union increases. Labor cost ↑, thus $SRAS$ ←.
    - Cost of other factors of production
      - Oil price increase due to a geopolitical conflict in the middle east. The cost of production ↑, thus $SRAS$ ←.
      - By the way, oil price change is a classical "shock" liked by many macroeconomists, because it is relatively exogenous to the economic system. Easier to tease out the causal relation, avoiding the endogeneity problem.
    - Productivity
      - A **temporary** boost in productivity. $SRAS$ →. Since it is a temporary swing of productivity, it does not change the long-run supply capacity. Thus $LRAS$ stays the same.
    - Capital
      - Capital stock damaged in the war, $SRAS$ ←.
      - Capital dropped due to a natural disaster, say an earthquake. $SRAS$ ←.
      - The change in capital stock, namely investment, however, is a *demand side* change, although it is a decision made by firms.

  - Long-run AS
    * A **permanent** increase in productivity due to techonological progress $LRAS \rightarrow$.
    * Infrastructure improvement increases productivity. $LRAS \rightarrow$.
    * The education level of labors improve. $LRAS \rightarrow$.
    * Change in steady state capital stock level k^* due to
      · Change in capital share in the production function. α in the Cobb-Douglass function $Y = AK^{\alpha}L^{1-\alpha}$.
      · A change in saving rate, which is implicitly affected by the preferences of the people in the country. For instance, East Asians are said to be very thrift and like to save money.

- Aggregate demand
  - Any factors that affect one of the following components of aggregate expenditure: consumption C, investment I, government spending G and exports/imports NX, have impacts on the demand side of the economy. (A shift of AD curve.)
  - First, the aggregate demand curve is downward sloping. Why?
    * Not due to **substitution effect** across goods. This is the change in overall price.
      · But can be due to intertemporal substitution effects. (Substitution between today and tomorrow. But not important for undergrads macro.)
      · Higher price today means a higher real interest rate. This makes it more difficult to borrow to consume today and more attractive to save for tomorrow. $C \downarrow$, then $AD \downarrow$.
    * Due to **wealth effect**.
      · Nominal financial assets. Lower price level leads to real wealth increase, thus spend more.
    * Or **interest rate effect**.
      · An increase in the price level requires firms and households to demand more money to buy goods and to invest. This means selling financial assets in the market, which drives up the interest rate, dampening the aggregate demand.
  - Consumer
    * Consumer confidence increase. $C \uparrow$, then $AD \rightarrow$.
    * Expected future income or permanent income increase. $C \uparrow$, then $AD \rightarrow$.
    * Tax cut. Disposible income $\uparrow$, then $C \uparrow$, then $AD \rightarrow$.
    * Higher MPC due to easier access to credit, i.e. loosening borrowing constraint. $C \uparrow$, then $AD \rightarrow$.

    - Wealth effect, i.e., higher financial wealth due to the stock market boom leads to more consumption. $C \uparrow$, then $AD \rightarrow$.
    - Lower interest rate, cheaper to apply for a credit card or morgage loan by households, $C \uparrow$, then $AD \rightarrow$.
  - Firms
    - Lower interest rate. $I \uparrow$, thus $AD \rightarrow$.
    - Investment tax subsidy increase. $I \uparrow$, thus $AD \rightarrow$.
    - Investment tax cut. $I \uparrow$, thus $AD \rightarrow$.
  - The government
    - War spending or goverment spending increase. $AD \rightarrow$. (But this ignores crowding-out effect.)
    - Unexpected fiscal transfer, i.e. consumption voucher or tax rebate. $AD \rightarrow$.
    - Tax cut. Disposable income increase. Thus $AD \rightarrow$.
  - External sector
    - Weak global economy, i.e. the U.S. wants to buy less toys from your country, then net exports $\downarrow$, then $AD \leftarrow$.
    - Exchange rate depreciation. Your exports become cheaper to foreigners, thus, net exports increase. $AD \rightarrow$.

- A few confusing cases.
  - Investment is in demand side of the economy. But by changing the capital stock, it may change the supply side of the economy in the future.
  - Infrastructure investment, in the short run, has demand effects. For instance, it is typically financed by government spending. But in the long run, it may also change the supply side. By providing better public service, firms are more productive, thus long-run aggregate supply increases.

2 Adjustment of the economy after shocks

It is important to understand the adjustment path of the economy following shocks in an AS-AD model.

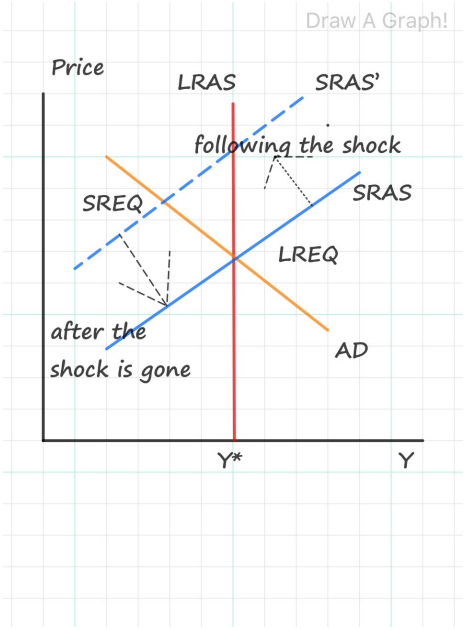
We consider two possible shocks. One is a negative supply shock, i.e. temporal oil price/supply chain disruption. The other is a positive demand shock, i.e. higher sentiment/government spending.

As shown in Figure 1, a temporary negative supply shock shifts the SRAS to the left, bringing the economy to the SREQ (short-run equilibrium) left to the LREQ. The economy experiences a price increase (inflation) and less output (recession), thus a higher unemployment rate. When the negative shock is gone,

the SRAS **shifts back** to its original position, bringing the economy back to the old LRSQ. Notice there is no change in AD curve throughout the adjustment.

As shown in Figure 2, a temporary positive demand shock shifts AD to the right AD', bringing the economy to the new SREQ right to its LREQ. A higher demand causes both price and output to rise, thus lowering the unemployment rate. While this happens, workers start asking for a higher wage, and this causes the SRAS to shift left and to intersect with the AD' where the natural output Y^* is., and the economy arrives at the new long-run equilibrium LREQ' with a higher price. Notice here, unlike supply shocks, the LREQ moves to a different price level.

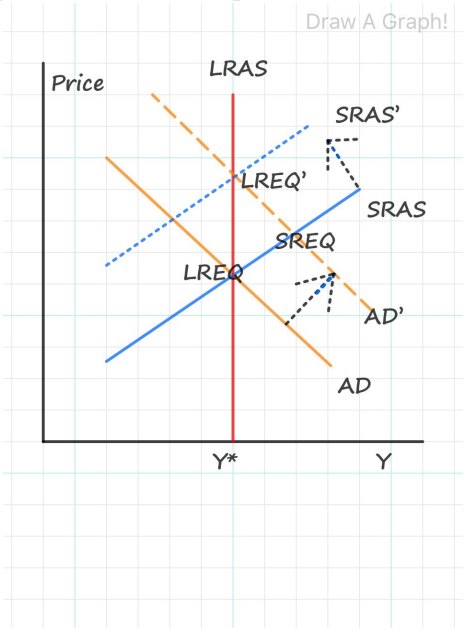
Figure 1: AS-AD after a temporary supply shock



3 Connections between Aggregate Expenditure Model, Loanable Fund Model and AS-AD Model

- Aggregate demand curve and aggregate expenditure
  - AD curve describes the relationship between the price level and output. But there is no price in the AE model. Where is it exactly?
  - The change in price shifts the AE line in the AE model up/down, leading to a different Y^*. This gives a downward-sloping AD curve. (See Figure 2.)

captionAS-AD after a positive demand shock



- Aggregate demand and lonable fund model.

  The loanable fund model is used to analyze the relationship between the real interest rate and borrowing/lending by households to firms and governments. Consider the following example.

  - The government spending increases after a project of building more schools. Therefore, the demand for government funds $\rightarrow$ in the government diagram in the loanable fund model, leading to a higher real interest rate of government bonds. (See the upper right panel of Figure 3.)
  - This will lead to a left shift of the supply curve of funds to corporations in the corporate diagram, crowding out the private investment due to the higher interest rate. (See the upper right panel of Figure 3.)
  - Government spending increases should shift right the AD, but at the same time, another component of AD, investment I dropped due to the crowding-out effect. Therefore, whether AD shifts up or down depends on the relative change of G and I. (See the bottom graph of Figure 3.)
  - In the long-run, consider that building schools will lead to permanent improvement of education level. thus long-run productivity. This will also shift $LRAS$ to the right. (See the bottom graph of Figure 3.)

Figure 2: AD curve and AE model

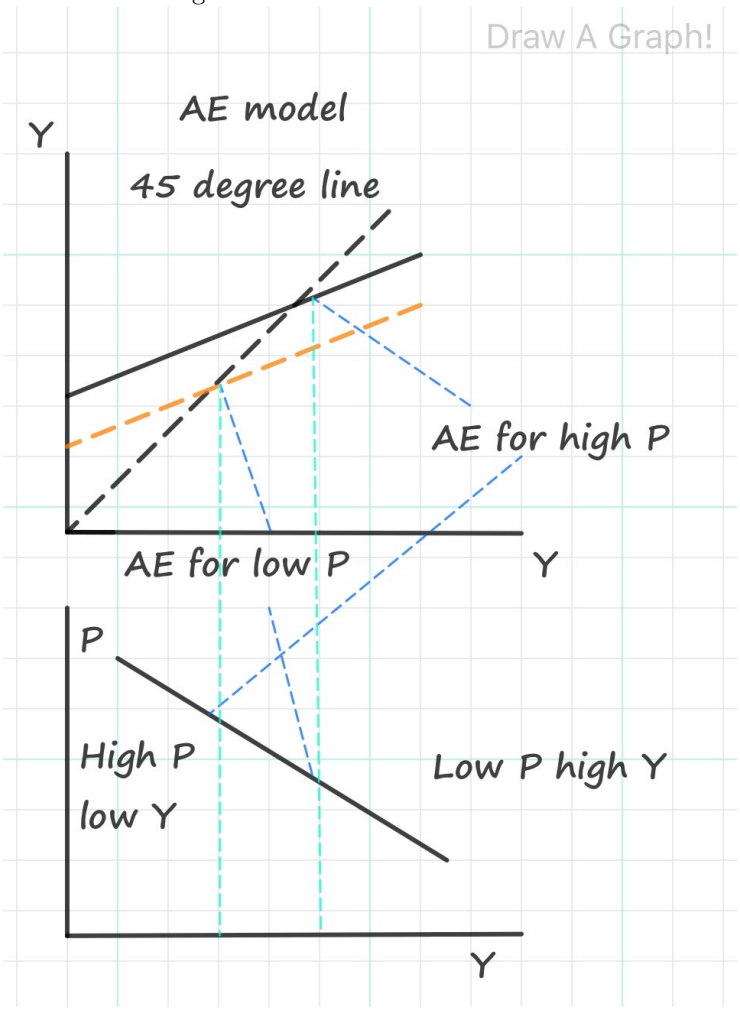
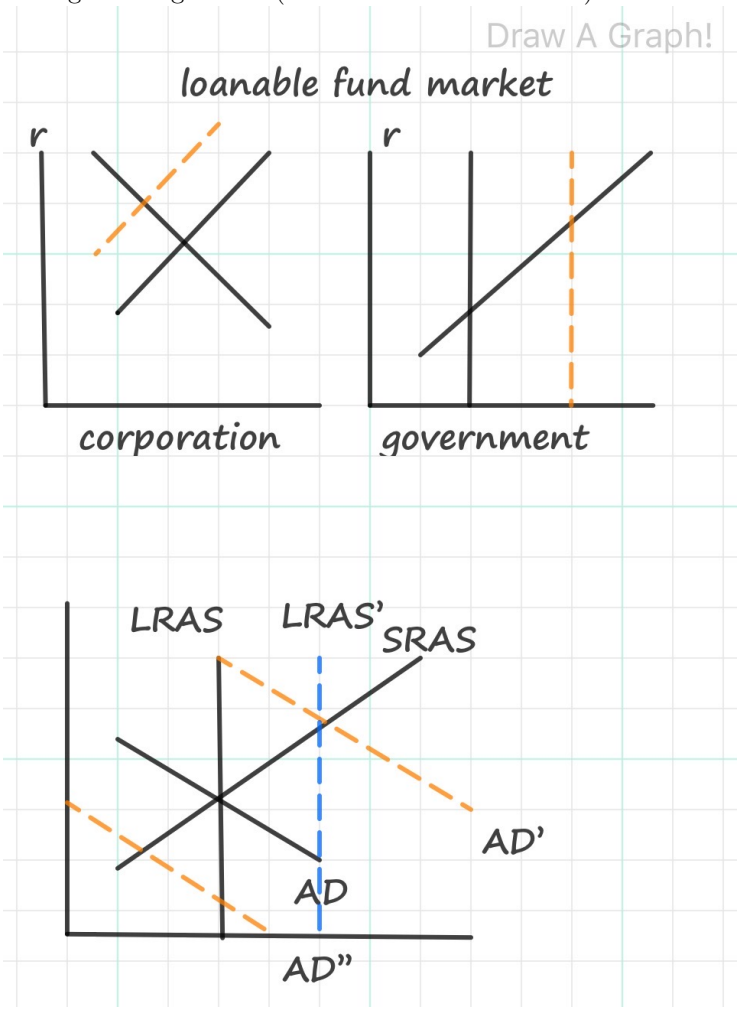


Figure 3: The short-run and long-run impacts of a sudden increase in government spending building schools(or same for infrastructure)



Interest Rates, Saving/Investment and Loanable Fund Model

Tao Wang

April 4, 2022

1 Interest Rate

Let us crash the differences between three concepts: interest rate, coupon rates and yield.

- A real-world example.
  - A firm issues a bond with the face value of \$100. It is one piece of paper. The face value of the bond is written on that piece of paper.
  - The maturity of the bond is 1 year, which is written on that piece of paper.
  - The coupon rate is, say 2%, which is also written on the paper.
  - What is not written on the bond is the price at which the bond is being traded in the market.
  - Let us assume that the firm can only sell the bond at \$90, because the market investors somehow believe this bond is not worth \$100.
  - Then next year, the bond matures, the face value of the bond is \$100 and the interest payment being \$100 $\times$ 2 % = \$2. Then in total, you earn \$102.
  - Your yield is equal to

$$\frac{102-90}{90} * 100\% = 13\%$$

    .
  - Notice this is way higher than the coupon rate 2%. Why? you bought the bond at a cheaper price than its face value.
  - This is also why the yield can be negative, i.e. you buy a bond at \$103 and get \$102 next year at maturity.
  - Zero coupon: a bond that promises to pay you the face value at maturity.

  - As a general principle,

$$\text{Yield} \neq \text{coupon rate}$$

  - Price and yield.

$$\text{Price} \uparrow \leftrightarrow \text{Yiled} \downarrow$$

  - Now interest rates. There are many interest rates in the market. The yield of this firm is just one of those. And with no-arbitrage, the interest rate with the same maturity and same risks should be approximately equal.

$$\text{interest rate} \supset \text{yield of this bond, deposit rates, etc.}$$

- As far as we are concerned, the interest rate is the "price" of the money.
  - The price of the good is the price.
  - The price of the labor is wage.
  - The price of the money is **interest rate**.
- Supply and demand of the money.
  - Who supplies the money? Lenders.
    - Households lend money, either directly, or indirectly to borrowers.
  - Who demands the money? Borrowers.
    - Firms may borrow money.
    - Government may borrow money.
  - The form of borrowing and lending.
    - You deposit money in the banks and banks issue the loan to the borrowers.
    - Invest in the bonds from in the market, either directly or indirectly through your mutual funds.
    - Buying stocks, in a broad sense. But there is an important difference between equity and debt in terms of the investors' claim to the rights in case of default.
  - Determinants of the **real** interest rate.
    - Duration (maturity).
      - The longer the bond is to mature, the more likely something happens in between. Investors need compensation for bearing this risk.
    - Default risks.

- The more likely the debt is not paid back, the higher the default risk is.
- The rule of thumb: government bond is less risky than profitable and big corporations than small/unknown/new firms.

- Nominal and real interest rate.

  Ex post:
  $$i = r + \pi$$
  Ex ante:
  $$i = r + E(\pi)$$
  The latter is in investors' minds when they make lending decisions.

2 Saving and Investment

- Whatever is not consumed is saved.
  $$S = Y - C - G$$
- According to expenditure approach of GDP
  $$I = Y - C - G$$
- Therefore, $S = I$.
  - This has to hold no matter if the market is in equilibrium or not.
  - It is a equity in flow, not in stock.
- Why exactly?
  - When you put the money in the bank, i.e. saving, this money is used to finance investment by firms.
  - Remeber the investment has two parts in it. Planned investment and unplanned investment, i.e. inventory.
    $$S = I_{up} + I_p$$
    The composition of I_{up} and I_p may change, but the total amount is equal to saving.
- A heads-up on an open economy.
  $$S - I = \underbrace{\text{Net exports}}_{\text{Current Account Surplus}}$$

3 Loanable Fund Model

- Two groups of people (or entities) in the economy.
  - People who do not need to spend money now.
  - People/firms want to borrow money.
- In the real world, the saving is converted to investment through financial institutions.
- In order to describe such a market, we go back to supply and demand again.
  - Vertical axis: real interest rate, the price of the funds.
  - Horizontal axis: the quantity of the funds.
  - The supply curve (from those who lead, i.e. households). Upward sloping because a higher real rate makes it more attractive to lend money instead of spending it today to households.
  - The demand curve (from those who borrow) vary across firms/governments
    - Government demand curves are vertical, because no matter how expensive of the borrowing cost is, the government spending needs to be financed.
    - Firms' demand curves are downward sloping because a higher rate makes it more costly to borrow.
  - Two sector model: government borrowing and corporations.
    - We use the two-sector model to capture the compositional change of private lending and public lending.
    - The key assumption: total available funds to be supplied by the households are constant.
    - Why aren't the rates in two markets equal? Because corporate bonds are riskier.
    - Some scenarios to consider.
      - **Crowding-out Effect**. A natural disaster calls for an emergency outlay. Demand for government bond $\rightarrow$, equilibrium rate $\uparrow$. Funds lent to the government $\uparrow$. Funds to the firms $\downarrow$ for a given real rate. Supply curve in the corporate bond market $\leftarrow$. Lending to firms $\downarrow$ and the rate $\uparrow$.
      - **Higher risk appetite.** A nationwide increase in household risk appetite. Given the same amount of funds households are willing to supply, some funds flow from government bonds to corporate bonds market. Supply curve $\leftarrow$ in G market and $\rightarrow$ in C market. Higher lending and lower rate in C and Lower lending and higher rate in G. The risky premium shrinks.

- **Weak demand.** Weak economic confidence lowers consumption, thus increases the total amount of available funds. Supply curves $\rightarrow$ in both C and G markets. Equilibrium rates are lower in both markets.
- **Weak investment.** Firm suddenly takes fewer investment projects due to the uncertainty associated with corporation tax reforms. Demand for funds by corporations shift $\leftarrow$. Lower rate and lower lending in C. Supply for the funds to $G \rightarrow$.
- **Increased inflation expectation.** A sudden change in inflation expectation. Everyone thinks inflation will be 4% instead of 2% next couple of years. There will be no changes in the graphs of the loanable fund model! Because the real rates that matter for your decisions remain the same.

Money and Money Policy

Tao Wang

April 18, 2022

1 Two views of money

- Two views on the roles of the money and how monetary policy (MP) is implemented.
  - **Quantity** view. The central bank sets the growth rate of the money supply according to the formula below.

$$\underbrace{P}_{\text{Price level}}\ \underbrace{Y}_{\text{Output}} = \underbrace{V}_{\text{Velocity}}\ \underbrace{M}_{\text{Money supply}}$$

    The intuition behind the equation is this. Money is used for the transaction. The size of the transaction depends on the total monetary value of the economy, i.e. the price times quantity PY. At the same time, the faster each dollar note changes hand from one person to the other, or is used the second time, i.e. higher V, the less amount of money is needed to play the role.

    We know that the growth rate of a multiplication of two components is the summation of growth rates of two, so we have the following formula.

$$\%M + \%V = \%P + \%Y$$

    If V is a constant, $\%V = 0$, then the monetary policy is to set the growth rate of the money supply to be the summation target inflation rate and growth rate.

    This was Milton Friedman's view.

  - There are several problems with the quantity view.
    - **Problem 1.** The problem is that V is not stable.
      For instance, a one-dollar bill changes hand very fast in an economic boom but very slow in recessions. For instance, you spend the dollar you earn today on tipping the waiter at the restaurant, who immediately spends the dollar on grabbing a beer with his friend.

* **Problem 2.** And the growth rate of M is hard to control in practice. The central bank does not have perfect control over the money supply since it is thousands of commercial banks who really decide how much "money" is used in the market.
* **Problem 3.** Besides, the definition of the money we should target is not even clear. Only cash in circulation? Or checking accounts? Also saving accounts? And money market fund? Since digital currency is used as transactions, should we include them as well?

2 Money Multiplier

- Commercial banks and the central bank.
  - We have known that the open market operation is that the Federal Reserve buys and sells treasuries from the market.
  - Who do they buy and sell with? *Commercial banks.*
  - Below is a commercial bank's balance sheet.
  - An expansionary(contractionary) monetary policy move, i.e. the central bank buys(sells) T-bills in the market from commercial banks. Therefore, the commercial bank has an increase (decrease) in reserve assets, decrease(increase) in T-bill assets.
  - What is the reserve? Basically the cash.
    * **Required reserve**. The bank regulators require the commercial banks to hold some cash in hands in case there is trouble. i.e. people line up to withdraw the money altogether.
    * The rest, **excessive reserve**. The banks can lend all the money out (to make money).
    * Obviously, the lower the required reserve ratio (RRR) is, the bigger the proportion of the cash can be lent out.
    * This determines how much money is created in the economy.
    * But how much exactly money can be created by this?
  - Money multiplier.
    * Imagine you have \$100 total reserves.
    * 20% of them is required to be held by the bank.
    * Thus \$80 can be lent out.
    * Each one dollar of this \$80 can be lent to the first person, which can be used by the next person, the next person, etc.
    * So the total amount of money the excessive reserve can create is

$$MMP = 0.8 + 0.8^2 + 0.8^3 + ... + 0.8^\infty = \frac{1}{1-0.8} = 5 \equiv \frac{1}{RRR}$$

Monetary Policy in Loanable Fund Model, and Taylor Rule

Tao Wang

April 18, 2022

1 Two views of monetary policies (continued)

- Unlike the **quantity** view, the **price view** of the role of money and monetary policy states that it is the interest rate, the price of the money, that affects the decisions such as borrowing and lending of firms and households and the central bank changes the trajectory of the economy by influencing (notice the word being "influence" not directly "set") this interest rate, instead of controlling the quantity of the money supplied to the market, as perceived in the quantity view.

- The action central bank takes to influence the interest rate in the economy is called open market operation. [1] Basically, the central bank and commercial banks buy and sell government bonds between them. The government bonds can be both the short-term bonds, which is called T-bills, and the long-term government bonds, called T-notes. By changing the supply of the funds to the government bond market, it directly affects the interest rates in the government market and the effect will be transmitted into corporate bond markets.

- This logic behind the mechanism is the best characterized in the loanable fund model. (See Figure 1.)

  - A monetary policy easing means the Fed buys the short-term T-bills so that the demand by government for household funds are less. This leads to a left shift of the vertical demand line in the T-bill market.
  - Since now less loanable funds by households are supplied to T-bills, the supply curve of funds in the long-term government bond market and corporate bond market both shift to the right.
  - As a result. the equilibrium real interest rates in three markets are all lower than before.

[1] This is also why, the body which decides the monetary policy in the United States is called the Federal Open Market Committee (FOMC).

Figure 1: How does monetary policy easing work?

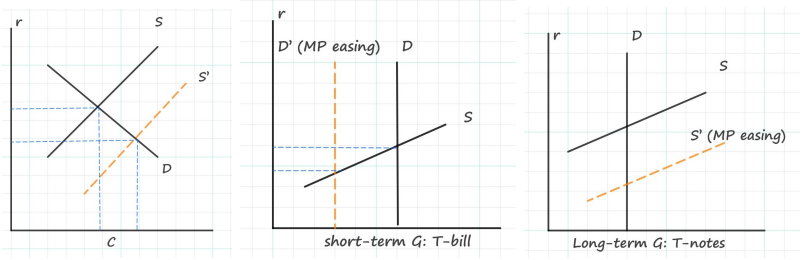
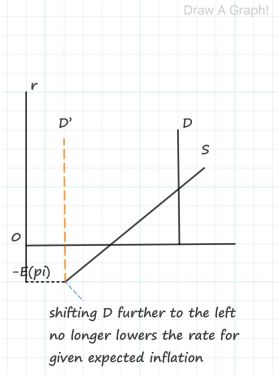


Figure 2: Zero Lower Bound



- Traditionally, Fed only operates in the short-term treasury security market, i.e. T-note markets. But there are limits to this policy, as it is obvious after final crisis in 2008, the most notable of which is the zero lower bound (ZLB) of the nominal interest rate. For instance, if the nominal rate of T-bill cannot go below zero, the real rate cannot be lower than $-E(\pi)$, negative inflation expectation. (See Figure 2).

  In responses, two sets of non-conventional monetary policy were pursued in addition to what we described above.

  - **Quantitative Easing (QE):** instead of only buying T-notes, the Fed also bought T-bills, i.e. the long-term government bonds.
  - **Forward Guidance:** At the ZLB, the effective lower bound of real interest rate is negative inflation expectation $-E(\pi)$. This means that another non-traditional monetary policy is to increase expected inflation $E(\pi)$, therefore, lower the effective real interest rate $-E(\pi)$.

2 Taylor Rule

- We have known that the monetary policy in the U.S. is to set a target range for the short-term interest rate.
- We have also known that the Fed is trying to achieve maximum employment, stable price, and moderate long term interest rates.
- The question is then how exactly Fed determines the "appropriate" federal fund rate in response to inflation and unemployment under different conditions of the economy.
- It turns out that the Fed has implicitly followed some pattern when making decisions, as summarized by Taylor rule. It summarizes a historical relationship between the federal funds rate target set by the Federal Reserve, inflation and unemployment gap (or output gap). It is written as below.

$$ff = \underbrace{r^* + \pi}_{\text{neutral federal fund rate}} + \alpha \underbrace{(\pi - \pi^*)}_{\text{inflation gap}} + \underbrace{(u^* - u)}_{\text{unemployment gap}}$$

  where

  - ff is federal funds rate.
  - r^* is the **natural real** interest rate.
  - π is the inflation.
  - π^* is the inflation target.
  - u is the unemployment rate.
  - u^* is the natural rate of unemployment.

- Qualitatively, the rule states the following responses.
  - Federal funds rate increase with inflation. Specifically, when the inflation is **too high**, exceeding the inflation target, $\pi > \pi^*$, the federal funds rate should be higher than the neutral rate, $r^* + \pi$.
  - Negatively responds to unemployment rates. If this is too low compared to the natural rate, $u < u^*$, the federal fund rate should be set higher than neutral rate, $r^* + \pi$.
- The meaning of α. The bigger α is, the more weight is given by the fed to inflation compared to the unemployment rate.
  - This we call "hawkish".
  - The opposite is called "dovish".

- Plugging in some numerical values.
  - $\pi^* = 2\%$
  - $u^* = 5\%$
  - $\alpha = 0.5$
  - $r^* = 2\%$
- Then the neutral rate is

$$ff = 2\% + 2\% = 4\%$$

  This means any rate is above (below) 4% is expansionary(contractionary).

- Important to know about Taylor's rule.
  - It is a summarized pattern based on historical data and Fed's practices.
  - But it is not necessarily the **right** rule Fed should follow.
  - Therefore, in its nature, it is positive, not normative.
    - Positive means descriptive and empirical. It concerns what "is", "was", or "will be", and contains no indication of approval or disapproval (what should be).
    - Normative is the opposite. It is about what is good and what is bad, what should be done, etc.

- Why do we need a rule in the first place?
  - Rule-based policy leads to predictability, helps stabilize expectations, builds credibility and discipline policymakers from making arbitrary decisions.
  - Discretionary policy involves more contingent decisions based on specific considerations of the circumstances. It has more flexibility but subjects the policy to arbitrary judgments.

Phillips Curve

Tao Wang

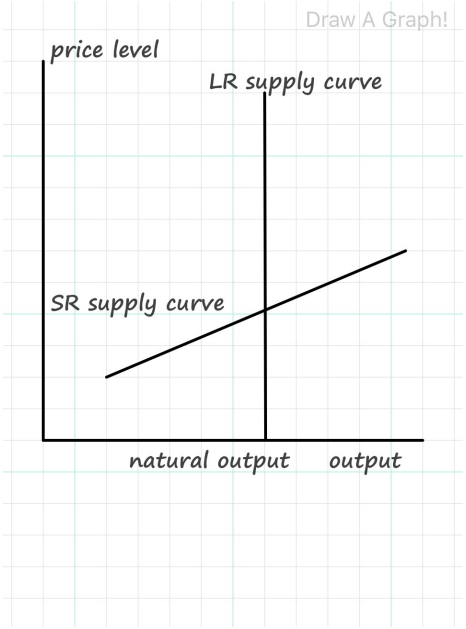
April 25, 2022

1 Phillips Curve (PC)

Forget about the PC for a moment, let us start by understanding the **aggregate** supply of the economy.

- How does aggregate supply depend on the price? (See Figure 1)

Figure 1: Short-run and long-run aggregate supply curve



  - The vertical axis is the price level. (See Figure 1.)
    - Overall price level of the economy, instead of the price of a particular good.
  - The horizontal axis is the output level. (See Figure 1.)

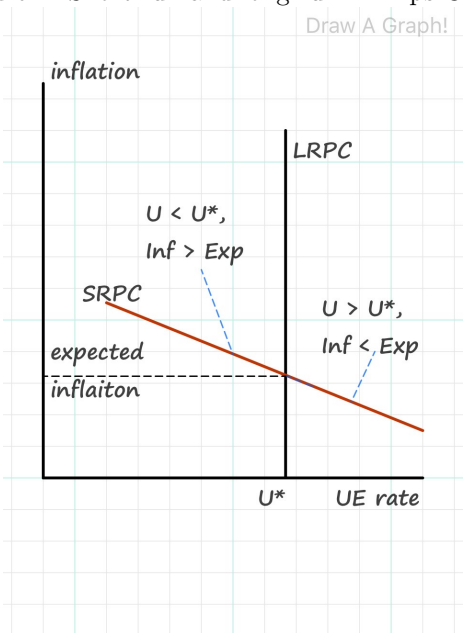
  - In the long-run, the supply curve is vertical. Why?
    * **Only relative price matters.** If everything becomes twice as expensive as now, there is no relative price change. All firms will produce exactly the same. Only relative price matters for economic decisions.
    * **Price does not affect the "real" side of the economy.** The supply of the economy, i.e. how much you can produce, fundamentally depends on the productivity, the number of labors and capital, etc.

- Short-run: upward supply curve. Why?
  - **The most simple reason**. When you want to produce more, you need to hire more workers (lower unemployment rate). This competition bid up the wage. In order to compensate for the increase in labor cost, you need to also increase the price (higher inflation).
  - There are various underlying assumptions for this to happen.
    * **Assumption 1: sticky wage**. The goods prices increase in response to higher demand, while the wage and the cost of production have not risen yet. Therefore, firms find it profitable to increase production in response to higher prices.
    * **Assumption 2: sticky price**. In the real world, the price may not change simultaneously. You may be able to sell your goods at a higher price without seeing your cost of production increase.
    * There are many mechanisms behind this upward sloping curve. Although economists disagree on which one is the major reason, they seem to all agree that in the short run the aggregate supply curve is indeed upward sloping.

- Phillips Curve (PC).

  Corresponding to the short-run and long-run aggregate supply curve (Figure 1), we have a short-run and long-run PC. (See Figure 2). Remeber that PC is all about the supply side of the economy.
  - Change the horizontal axis from the output to the unemployment rate. They are negatively correlated. Higher output, meaning a lower unemployment rate.
  - Change the vertical axis from price level to the inflation, i.e. the change in the price level.
  - Long-run Phillips curve is vertical for exactly the same reason why the aggregate supply curve is vertical in the long-run.
  - Short-run PC is downward sloping exactly for the same reason why aggregate supply is upward sloping.

Figure 2: Short-run and long-run Phillips Curve



- Formula of PC:

$$\pi = E(\pi) + \alpha(U^* - U)$$

  - U is the unemployment rate.
  - U^* is the natural rate of unemployment. or **NAIRU**: non-accelarating inflation rate of unemployment. Taken as given.
  - π is inflation.
  - $E(\pi)$ is expected inflation, taken as given at one point of the time.

- Intuitively, the logic behind the formula is the following
  - There is some natural speed of the growth rate that can be attained by the economy if all working-age people at brought to work, unless they are structurally and frictionally unemployed.
  - A higher speed than that (namely lower unemployment, to the left of LRPC) is somehow too fast, called overheating.
  - A lower speed than that (namely higher unemployment, to the right of LRPC) implies not reaching the potential of the economic growth one could have achieved. It is sometimes called "slack".
  - When the firm suddenly sees that the inflation turns out to be higher than what they think, suggesting there is stronger de-

mand, firms decide to produce more, therefore, lowering the unemployment.

* The intersection of short-run and long-run PC is expected inflation. Short-run PC shifts up implies higher expected inflation.

$$U = U^* \rightarrow \pi = E(\pi)$$

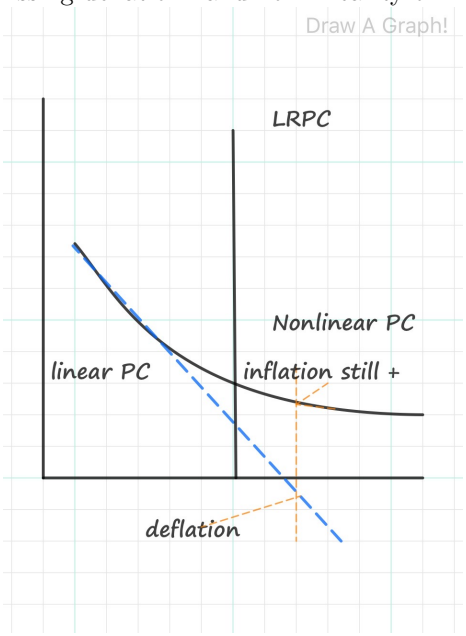
In different periods of history, the expected inflation level has been different. For instance, inflation was way higher in the 1970s than now.

* The natural rate of unemployment has been long taken as a constant, but recent evidence also suggests that U^* is lower than we thought. Maybe there is a fundamental change in labor markets.

2 Additional discussions PC

- Phillips Curve and monetary policy.
  * Macroeconomic policy can be seen as choosing one point in the short-run PC.
  * The central bank and government are faced with the trade-off between a lower unemployment rate and bearing higher inflation.
  * For instance, the Federal reserve bank has double mandates: attaining a sustainable level of employment and price stability.
  * Some central banks, in contrast, only state on the single mandate of price stability.
  * But the *divine coincidence* describes the case in which a single-goal central bank happens to achieve both mandates. Why?
    · According to the formula of PC, bringing inflation to the expected level also means that the unemployment rate attains to its natural level.
- Two current issues about PC.
  * **"Missing Deflation"**: in the aftermath of the financial crisis, the unemployment rate is so high that according to the Phillips Curve, there should have been negative inflation, namely deflation. But it was not the case. This suggests that Phillips is flatter at higher U, thus non-linear. (See Figure 3)
  * **"Missing Inflation"**: the recent two years 2018-2019, the labor market has become strong enough (U as low as 3%), suggesting, according to Phillips Curve, there should be a rise in inflation. But we have not seen it yet.
- Consider the following scenarios. What would happen to short-run or long-run Phillips Curve? Shift up or down? Move along? Or becomes flatter or steeper?

Figure 3: "Missing deflation" and non-linearity of Phillips Curve



- **Automation leads to a higher structural unemployment rate.** $U^* \uparrow$ Therefore LRPC $\rightarrow$. (See Figure 4.)
- **A "dovish" new central bank governor**. Dovish means he does not care about inflation being too high. $E(\pi) \uparrow$, SRPC $\uparrow$.
- **Stronger confidence and stronger market demand.** Then moving up along the SRPC, lower U and higher π.
  - These are forces that affect aggregate demand.
- **A government with little credibility just won the election.** The governments used to promise to people 2% inflation target, but now it obvious they will not fulfill its earlier promise anymore. $E(\pi) \uparrow$, SRPC $\uparrow$.
- **The rise of "gig economy"**. The flexible work arrangement from "gig" economy reduces the bargaining power of labors, because firms can fire/hire workers more easily. This means wage does not need to increase as much as before to hire workers. Therefore, SRPC becomes "flatter".

Reference

1. The Economist's special report on inflation. `https://www.economist.com/special-report/2019/10/10/inflation-is-losing-its-meaning-as-an-economic-indicator`

Figure 4: Higher natural rate of unemployment and LRPC

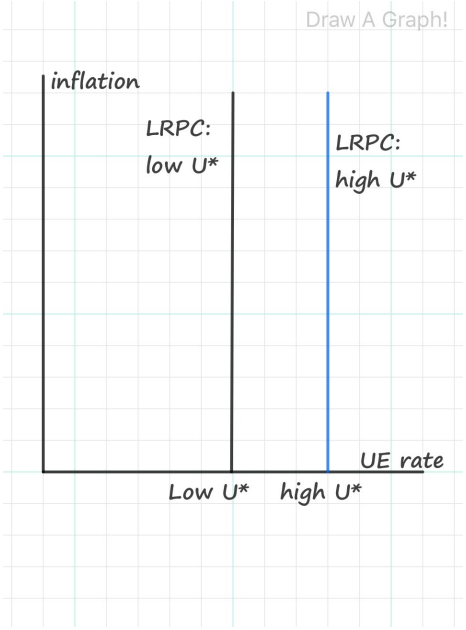
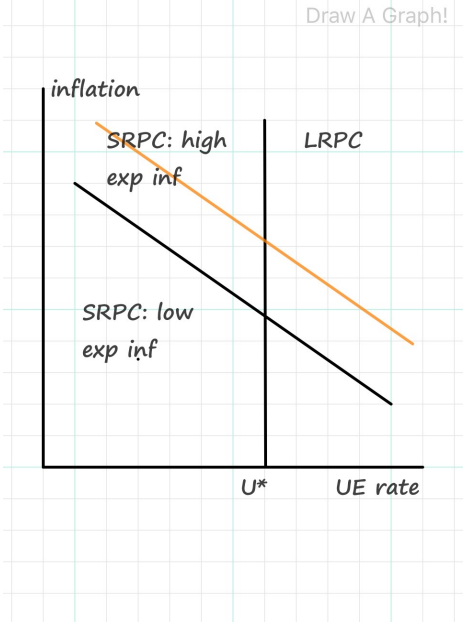


Figure 5: Higher expected inflation and SRPC



Fiscal and Monetary Policy

Tao Wang

November 29, 2021

1 Differences between fiscal and monetary policy

A new learner of macroeconomics may sometimes feel confused about the distinction between fiscal and monetary policy. Table 1 presents the major similarities and differences between the two.

1.1 Balance sheets of the government and the central bank

Another way to understand the relationship between FP and MP is to examine the items shown on the balance sheets of the treasury and the central bank.

- The treasury's balance sheet
  - Asset: cash, government-owned properties, and some loans issued by the government, such as student loans backed by the federal government.
  - Liability: debt outstanding, i.e. treasury securities.
- The central bank's balance sheet
  - Asset: treasury securities, mortgage-backed securities (MBS), foreign exchange reserves and gold.
  - Liability: the currency in circulation and reserves owned by the commercial banks but stored in the central bank. The central bank may also borrow from other central banks.

2 Monetary Policy

- First and foremost, the monetary policy is aggregate demand policy. It works through shifting *aggregate demand* (AD) curve, and moving along the short-run supply curve (SRAS). But it does not shift the long-run supply curve. (LRAS). Therefore, in the long-run monetary policy does not have real impacts of the economy.

Table 1: Monetary and Fiscal Policy

	FP	MP
Organization	Treasury	The Central Bank
Relationship with the Government	Always part of the government	Different types. Can be super-national, i.e. ECB, or can be legally independent from the administrative branch, i.e. Federal Reserve Bank system, or even private owned, i.e. Bank of England until nationalization in 1946, or part of the government like most in developing countries.
Common objectives	Managing aggregate demand	Managing aggregate demand
Specific objectives	Balancing government budget, wealth redistribution through tax, other specific public policy.	Stable price level (low inflation), low unemployment, and stable value of currency, etc.
Tools	Tax, government spending, government debt, government budget policy.	Interest rate, or money supply.
Mechanism via which it affects aggregate demand	1. Higher government spending directly increases AD. 2. Tax cut (increase) increases (decreases) disposible income of households, thus increases (decrease) consumption.	1. Expansionary (contractionary) MP makes it easier (more difficult) to borrow by firms, thus increases (decreases) investment. 2. Expansionary (contractionary) MP makes it less (more) attractive to save money, thus increases (decreases) consumption.
Impacts	Expansionary (contractionary) FP such as tax cut(increase) and increase (decrease) in government spending boosts (depresses) aggregate demand.	Expansionary (contractionary) monetary policy such as lower(higher) interest rate or an increase (decrease) in money supply boosts (depresses) aggregate demand.
Characteristics	The implementation takes time. More discretionary.	Timely and responsive. Tend to be more rule-based.
Other things to know	Auto-stabilizer items are not considered as fiscal policy. Discretionary spending not aimed at macroeconomic management is not fiscal policy either.	

Table 2: Balance sheet of commercial banks

Asset	Liability + Equity
T-bills	Deposits
Reserves	Equity
Loans	

- **Mandates of central bank** may differ. Some central banks only focuses on inflation. Some are concerned with inflation and unemployment at the same time. Some central banks, like those in emerging markets, also care about the value of their currency, i.e. exchange rate stability.
- **How MP works?** The central bank affects aggregate demand by influencing the short-term interest rate.