Session 1. Elasticity

Tao Wang

September 3, 2020

1 Elasticity

- Why do we need a concept like elasticity?
  - It is one of the most mportant concepts in economics.
  - Economics is very often about how the change in one thing is associated with/caused by the other thing.
  - More importantly, we want to make this relationship comparable between goods regardless of the units we are using to measure things. Elasticty measures this relationship free from the units.
- Definition
  - The elasticity of X with respect to Y is the **ratio** between the **percentage change** of X corresponding to one percentage increase in Y.
- Mathematics
  - Equation:

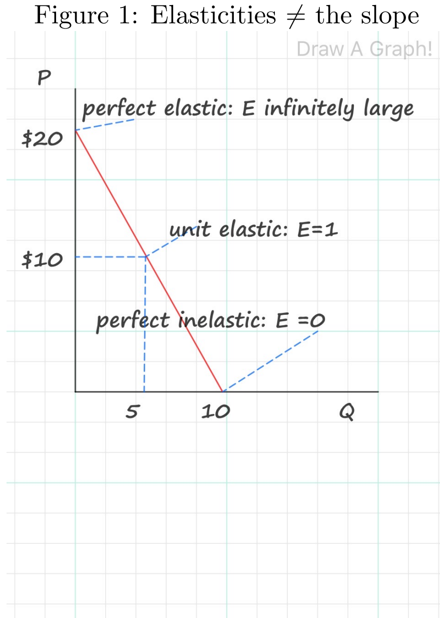
$$E_{XY} = \frac{\%\Delta X}{\%\Delta Y} = \frac{\frac{X'-X}{X} * 100\%}{\frac{Y'-Y}{Y} * 100\%} = \frac{\frac{X'-X}{X}}{\frac{Y'-Y}{Y}} = \frac{X'-X}{Y'-Y}\frac{Y}{X} = \frac{\Delta X}{\Delta Y}\frac{Y}{X}$$

  - Always remeber, the key word about elasticity is **percentage change** instead of **change**.
    - The elasticity does not just depend on changes in X and Y, but also on the initial levels of X and Y.
    - That means that the elasticity at different values of X and Y may be different.
- Why elasticity (percentage changes) instead of absoulte changes?

- $1 decrease in the price of the milk may lead one to buy 1 more bottle of milk or 0.5 more gallons of milk. We do not want the choices of measuring milk in terms of the bottle or gallon to affect our measure of how our demand for milk corresponds to the price changes. Notice in percentage change, they are the same.
- Also, a $100 change of price of a car is nothing but that of a toy car is something. You want to make sure you can still compare how sensitive you are to a price change of the two different goods by taking into account that they fall in entirely different price ranges.

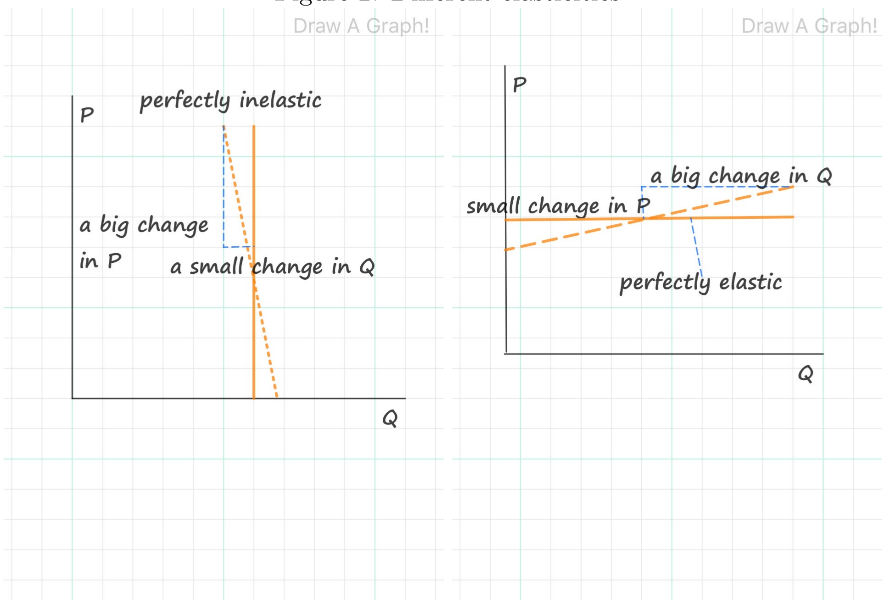
- Elasticity and the demand/ surpply curve
  - **The sign of elasticity**
    - Demand elasticty is negative if demand curve is downward sloping.
    - Supply elasticity is positive if the supply curve is upward sloping.
  - **The size of the elasticity**
    - Elasticity $\neq$ the slope of the curve
    - A linear demand or supply curve has the same slope everywhere, but it does **not** have the same elasticity everywhere. (See Figure **??**).

Figure 1: Elasticities $\neq$ the slope



- Different values of elasticity

Figure 2: Different elasticities



- Factors that affect elasticity
  - Demand elasticity
    * **Necessities versus non-necessities**: you have to buy food anyway (**low** elasticity), but you don't have to travel (**high** elasticity)
    * **Substituable or non-substitutable**: you can easily substitute a pizza for a bagel but you cannot substitute a particular textbook required by the class for another. (why is textbook way more expensive than a best-seller non-fiction?)
  - Supply elasticity
    * **Easily scale up the production or not**: Microsoft can trivially produce one more CD for Windows operating system, but the land cannot be expanded to build another resident building.
  - Both elasticity
    * **Short-run or long-run**: supply cannot adjust immediately because building factories and hiring workers take time. Demand cannot adjust in the short-run because people's taste does not change everyday.
- Other elasticities in economics

- **Income elasticity** of consumption: rich people will not spend more if they make 1% more but poor people consume more after 1% increase in the income.
- **Wage elasticity** of labor supply: people are willing to work a lot more hours (**high** elasticity), the same hours (**low** elasticity) or fewer hours (**negative** elasticity) if wage increase by 1%
- **Cross elasticity**: one buy more from the Brody coffee in response to a price increase in the Levering coffee (**positive cross elasticity**, i.e. substitutes) or one buy less food from the Levering dining hall (**negative cross elasticity**, i,e. complements) following the same change.

• Elasticities help explain many things
  - Why is textbook way more expensive than a regular book?
  - Why is it rare to see discounts on flight tickets at the last minute?
  - What does it suggest if people in decades ago and now both work for similar hours even though the wage has increased a lot?

Session 2. Analysis of the Market and Welfare

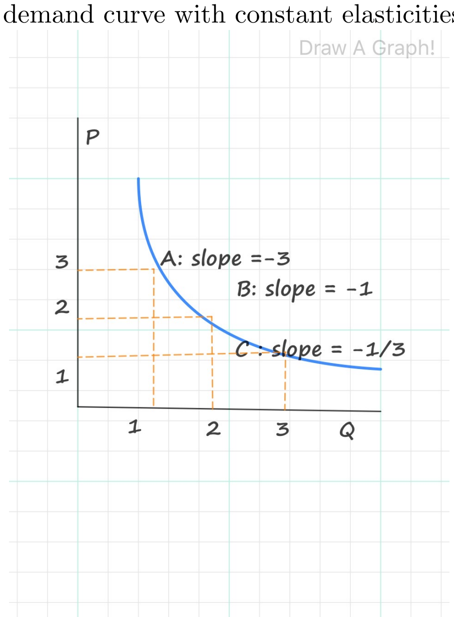
Tao Wang

September 10, 2020

1 Elasticity, part 2

- We know already that the linear demand curve/supply curve has different elasticities everywhere. But we could have a non-linear demand/supply curve of a particular shape with equal elasticities at any point of the curve. See Figure 1.
  - $E_A = \frac{\Delta Q}{\Delta P}\frac{P}{Q} = -\frac{1}{3} * \frac{3}{1} = -1$. The same for B and C. Therefore, all three points have the unit elasticity.

Figure 1: A demand curve with constant elasticities everywhere



- Income elasticitiy and the type of good
  - Income elasticity being **positive** means **normal good**: you want to buy more if you are richer.

- Income elasticity being **negative** means **inferior good**: you want to buy less if you are richer, i.e. junk food and staple food.
  - If the income elasticity is **extremely** negative so that the income effect dominates substitution effect, then the good is a called Giffin good.

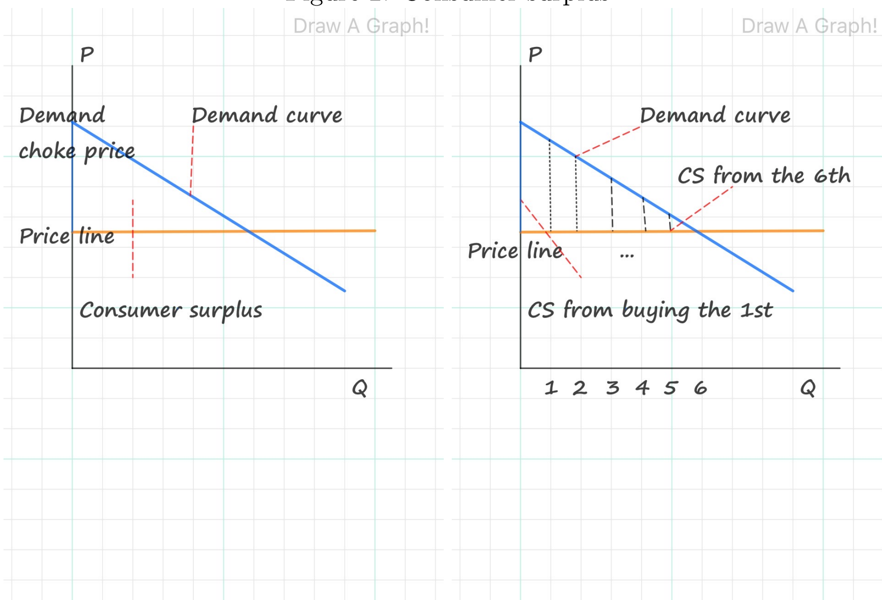
2 Consumer/Producer/Social Surplus

- What are the uses of these concepts?
  - We have understood how the market arrives at equilibrium. But how do we judge if the consumers and producers are satisfied with the market outcome?
  - We also need to evaluate if a certain policy/government intervention is really doing good to the society as a whole: **welfare improving**.
- Consumer surplus
  - Definition: CS measures the difference between the price a consumer is **willing to pay** and the price he/she **has to pay**.
    - Imagine you are a desperately thirsty traveler in the desert carving for a bottle of water. You vow to yourself: I can pay $200 to get that bottle. You finally found a vendor with the conscience and was offered a bottle by only $50. The extra joy you obtain from this is your "consumer surplus" from consuming that bottle of water.
  - Graphical representation: consumer surplus is represented by the area below the demand curve and above a price line. See Figure 2.
    - Remember that a consumer surplus is always associated with a particular price and quantity.
    - Spending money is always costly. But consumer surplus measures the satisfaction you would obtain **taking into account** that cost.
    - Why is it an area? We can think the area as a cumulative sum of the consumer surplus from each additional unit of the consumption brings to you. See the right plot of Figure 2.
  - Math:

$$CS = \frac{1}{2} \times Quantity \times (DemandChokePrice - Price)$$

    - **Demand choke price**: the price at which the demand is zero, or the intercept of the demand curve with the horizontal axis.

Figure 2: Consumer surplus



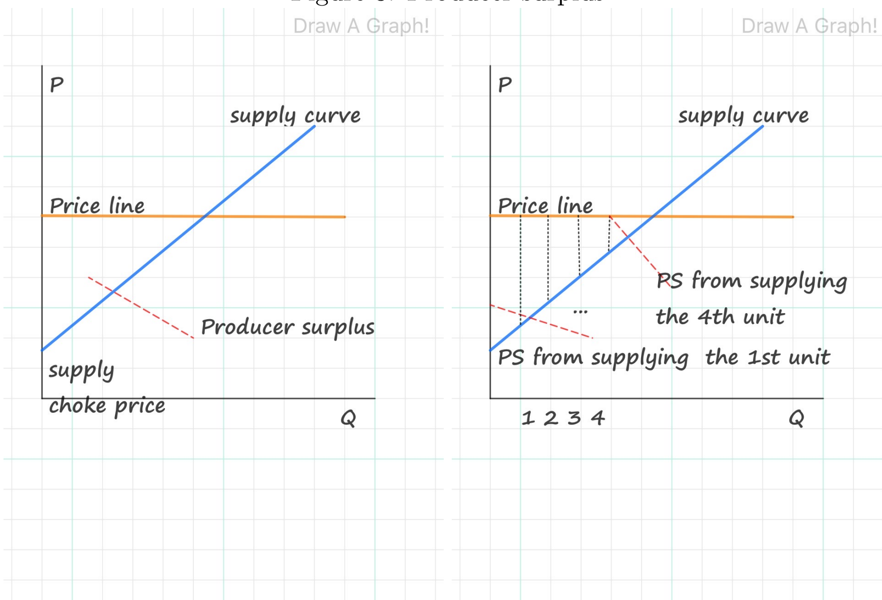
- The consumer surplus and demand elasticity: a **more inelastic** demand (a steeper demand curve) is associated with a higher consumer surplus (a bigger area below the demand curve and price line) at a given quantity.

- Producer surplus
  - Definition: PC is the difference between the price the producer **actually receives** and the **cost** of producing the good.
    - You can also think it as the difference between the price the producer receives and the **lowest price he/she would accept** so as to produce the good.
  - Graphical representation: PS is the area below the price line and above the supply curve. See Figure 3.
    - Similar to CS, you can think the area of the PS a cumulative sum of small trapezoids that represent the producer surplus you obtain from producing each additional unit of goods. See the right plot in Figure 3.
  - Math:

$$PC = \frac{1}{2} \times Quantity \times (Price - SupplyChokePrice)$$

    - **Supply choke price**: the price at which the supply is zero, or the intercetion of supply curve with horizontal axis.

Figure 3: Producer surplus



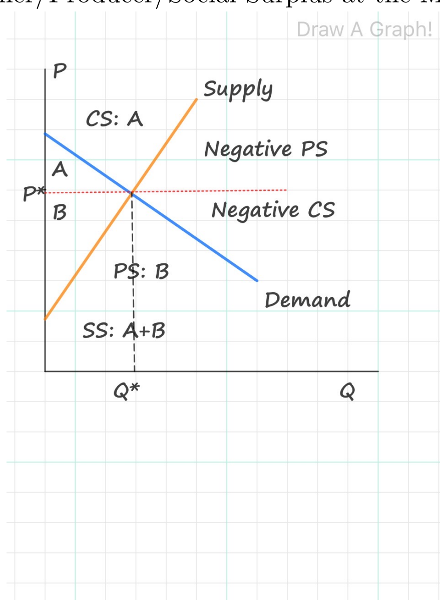
- The producer surplus and supply elasticity: a **more inelastic** supply (a steeper supply curve) is associated with a higher producer surplus (a bigger area above the supply curve and below the price line) at a given quantity.

- Consumer/producer/social surplus in the market equilibrium
  - Social surplus
    - Definition: the sum of CS and PS.
      - Intuition: when the producers sells the good at a price above the cost and the consumer buys the good at a price below the price she is willing to pay. Both are happier. The social surplus measures that.
    - Graphical representation. See Figure 4.

- Distribution of change in gains and losses from the change in market condition. (Next session).

3 Price/Qunantity regulations (Next Session)

Figure 4: Consumer/Producer/Social Surplus at the Market Equilibrium



Session 3. Analysis of the Market and Welfare, Part 2

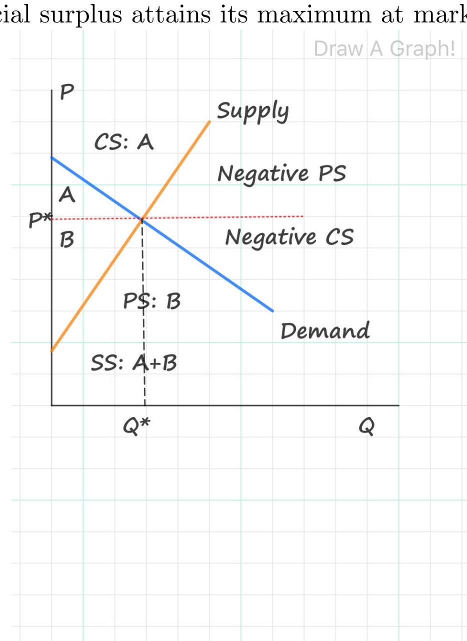
Tao Wang

September 17, 2020

1 Welfare at the Market Equilibrium

- Why does the social surplus attain its maximum under market equilibrium?
  - All consumers who are willing to pay a price higher than or equal to the market price have obtained the good. (See Figure 1).
  - At the same time, all the producers who are willing to sell the goods below the price have sold the good. (See Figure 1).

Figure 1: Social surplus attains its maximum at market equilibrium



- Distribution of change in gains and losses from the change in market condition.
  - A **right** shift in supply curve with fixed demand curve **increases CS** unambigously. It will increase PS if the demand is elastic and decreases PS if the demand is inelastic.
    - Intuition: a right shift in the supply curve means that the producer is willing to supply more for a given price. This leads to a higher quantity and lower prices. Since more consumers can buy the goods at a lower price, their welfare will rise unambiguously. This involves a transfer from the producer to the consumer. The PS will be **higher** if the benefits from **increse in quantity** exceeds the **loss from the decrease in price**. Whether PS increases or decreases depends on the relative effects of the two. The former scenario is more likely if the demand is **more elastic** (more horizontal).
  - A **right** shift in demand curve with fixed supply curve **increases PS** unambigously. It will **increase** CS if the supply is **elastic** and decreases CS if the demand is inelastic.
    - Intuition: (you fill it).

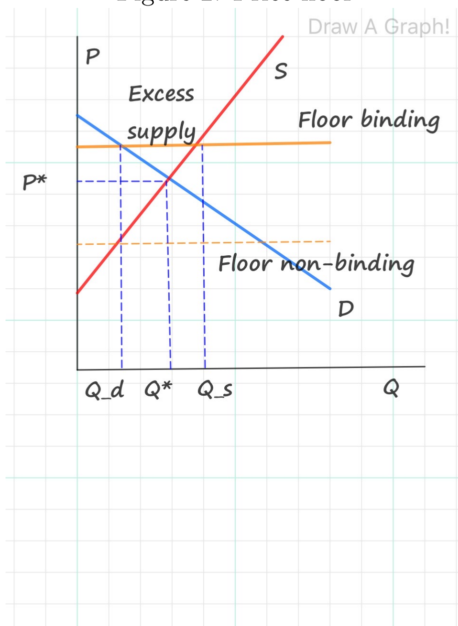
2 Welfare under price/qunantity regulations

2.1 Price regulations

- Price regulation takes forms of **price ceilling** and price floor. The former is meant to prevent prices from **increasing** and the latter for preventing prices from decreasing.
  - Price ceiling is usually "meant to help"[1] consumers, i.e. a box of sushi in the dining hall should not be too expensive to students; the health insurance should be affordable.
  - Price floor is usually "meant to help" producers, i.e. workers should at least earn the minimum wage; farmers should be guaranteed a minimum price of crops.
- Price regulations may not be always **binding**. Price floor is **not binding** when the government imposes a price **below** the market price, i,e. the minimum purchase price of crops during a drought.
  - Therefore, if binding, there will be **excess supply** under the price floor. See Figure 2. Producers still want to produce it but no one wants to buy it.

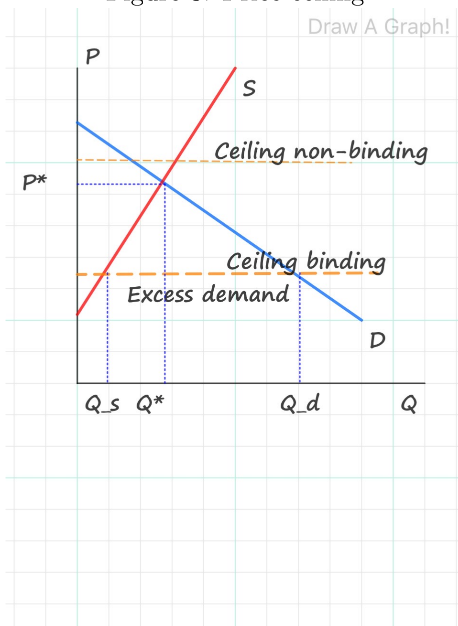
[1] Notice I said "meant to", implying that potentially this goodwill may not necessarily pay off as intended. What's worse, it induces social loss.

Figure 2: Price floor



- Price ceiling is not binding if it is above the market price, i,e. one roll of toilet paper during the pandemic should be sold below $100.
  - Therefore, if binding, there will be **excess demand** under the price floor. See Figure 3. Consumers still want it but no one wants to produce it.
- Price ceiling, when binding, induces loss to producers, unambiguously. But it may not necessarily help consumers it is typically meant to help, either See Figure 4. Under the price ceiling, producers sell less at a lower price, hence, it is a clear loss to them. A group of consumers who are lucky to get the good under the ceiling price is happier than when they pay the price in the market equilibrium. But another group of consumers who want to buy the good but unable to buy them, (because the producers no longer sell it to you) are worse off. Whether CS increases or decreases depends on the relative force of the two.
- The opposite could be said for price floors. See Figure 5. (You fill it)
- In both scenarios when the price regulations are binding, there is **unambigous social welfare loss**, deadweight loss. Because both policies prevent some transactions that would have taken place without the intervention.

Figure 3: Price ceiling



2.2 Quantity regulations

- In theory, quantity regulation could also take forms of both quantity cap (**quota**) or quantity floor. The former is more common, i.e. you cannot buy/sell more than 10 rolls of toilet paper. We also call this scenario **ration**. Let's focus on the quota from now.

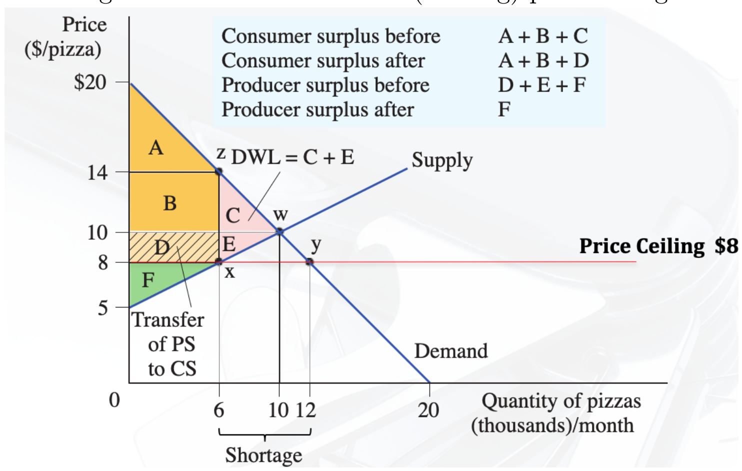
- These policies are typically enacted in the time of shortage, i.e. during wars and natural disasters.

- It is again not always binding. When it is binding, the quantity is **below** market equilibrium.

- If we consider the case in which the quota is imposed on producers, then the impacts of the quota are basically making the part of the original supply curve beyond the quantity allowed by the quota vertical (S' in Figure 6). The producer cannot supply more beyond the quota no matter what the price is, i.e. perfectly inelastic. The new intersection of supply and demand curve is associated with a higher price and lower quantity. See Figure 6.

- Such a regulation induces an unambiguous decrease in CS. As to the producer, since the producer is able to sell the goods at a higher price than the market price, this involves a transfer from the consumer to the producer. At the same time, however, since the producer can only sell fewer

Figure 4: Welfare loss under (binding) price ceiling



of the good, there is a loss in PS. This is why the quota may increase or decrease PS.

- Socially speaking, quota induces an unambiguous loss in society. The reason is simple. It prevents some transactions from happening which would have made both consumers and producers happier. See Figure 7.

3 Welfare under tax/subsidies (next session)

Figure 5: Welfare loss under (binding) price floor

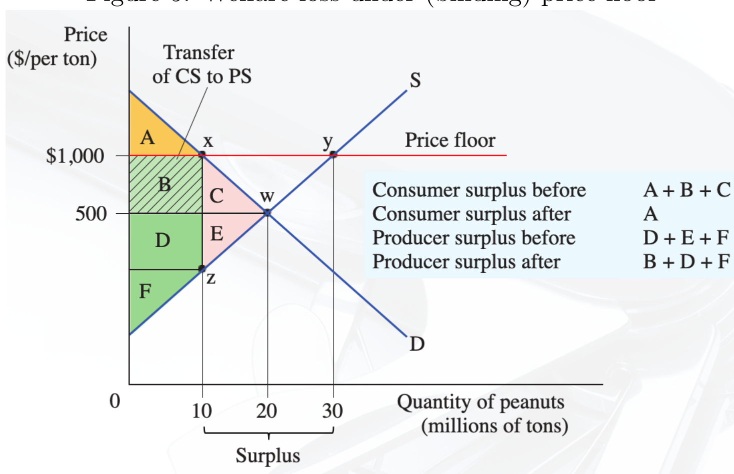


Figure 6: The effect of quota (on supply)

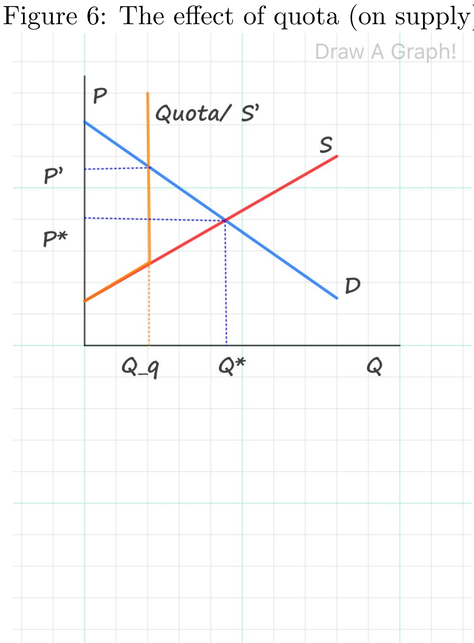
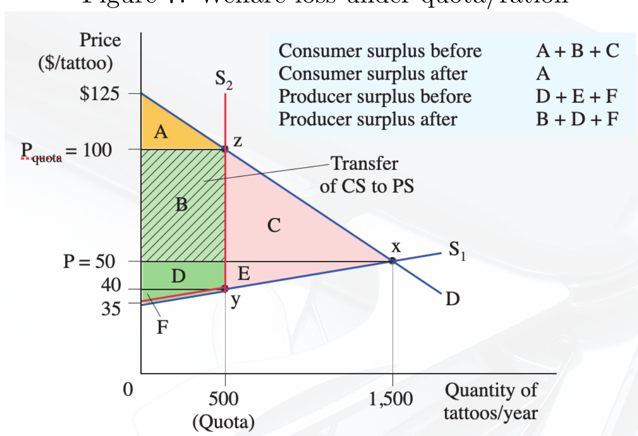


Figure 7: Welfare loss under quota/ration



Session 4. Analysis of the Market and Welfare, Part 3

Tao Wang

September 24, 2020

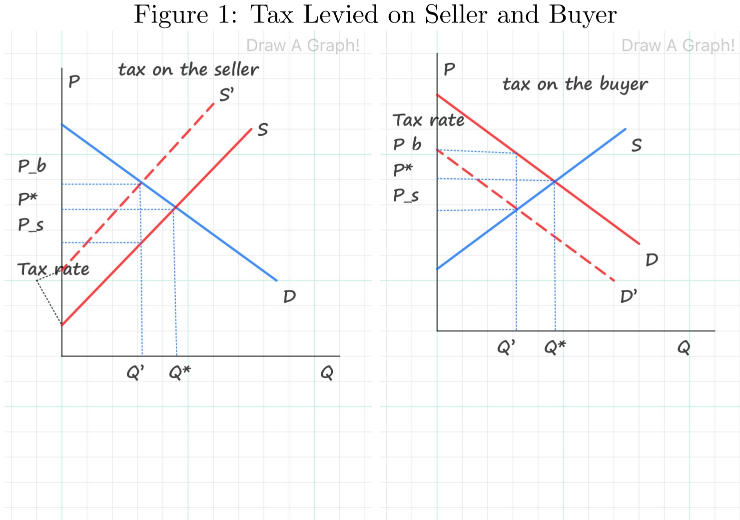
1 Welfare under Tax and Subsidy

1.1 Tax

- Why do we need tax?
  - Redistribution, i.e. progressive income taxes, wealth taxes, inheritance taxes, estate taxes.
  - Achieve other public policy objectives, i.e. soda tax, sin tax, sugary drink tax.
  - Addressing negative externality, i.e. pigovian tax, carbon tax.
  - Funding government expenditures, tax is the major source of government revenue, in addition to fees, tools and other revenue from selling public goods.
- Effects of tax
  - Higher price paid by the buyer and lower price received by the seller.
  - A wedge between the price the buyer pays and the price the seller receive, i.e. $p_b = p_s + tax$,
  - Lower quantity
  - Lower consumer surplus and lower producer surplus.
  - Deadweight loss
- How does the tax work?
  - Scenario 1. Tax on the seller. See left of Figure 1.
    * **A left shift of supply curve**: for a given quantity of supply, the price the seller need to receive higher by the size of the tax rate, i.e. the seller implicitly adds the tax he needs to pay in the new supply curve: selling 100 ice cream at \$2 before the tax, but can only do the same at \$2+tax now.

* **Lower quantity** The buyer is facing a lower supply. This leads to the lower quantity that is sold.
* **Buyer's price** Also, a lower supply leads to a higher price the buyer needs to pay.
* **Seller's price** However, the price that the seller would receive that truly goes to his pocket is not what the buyer pays, but the one after subtracting the tax. This is the price on the **old** supply curve corresponding to the new quantity of being sold. [1].
* **Lower CS** the buyer only buys less at a higher price (than the market price).
* **Lower PS** the seller sells less at a lower price (than the market price)
* **Deadweight loss** some buying and selling could have happened without tax is foregone.
* **Tax revenue** Tax rate times the new quantity is the tax revenue. A low tax rate with a higher quantity and a high tax rate with low quantity may end up giving you the same revenue.

Figure 1: Tax Levied on Seller and Buyer



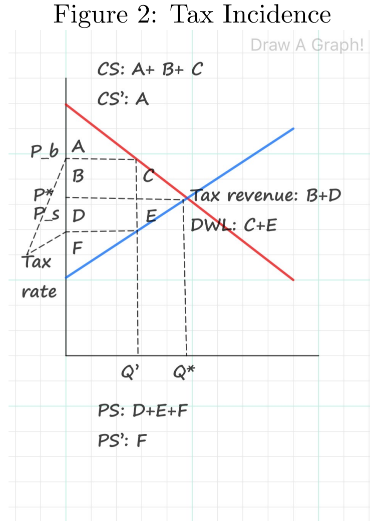
- Scenario 2. tax on buyers. See right graph of Figure in 1.
  * **A left shift of demand curve**: for a given qunatity of demand, the price the buyer is willing to take needs to be lower by the size of the tax rate, i.e. the buyer implicitly adds the tax he needs to pay in deciding his demand.

[1]Understanding why the supply curve facing the buyer is not the same as the one that really affects the seller's supply decision is important here.

* **Lower quantity** The seller is facing a lower demand. Less ends up being sold, a lower quantity.
* **Seller's price** The seller would receive a lower price because of lower demand.
* **Buyer's price** However, the price that the buyer have to pay that really goes out of his pocket is not the price the seller receives, but one adding tax to it. This happens to be the price on the **old** demand curve corresponding to the new quantity.
* The impacts on CS, PS and DWL are the same as scenario 1.

– Tax incidence. See Figure

Figure 2: Tax Incidence



* Turns out that scenario 1 and 2 leads to the same price and quantity after the tax for a same tax rate.
* Same tax revenue.
* Same tax burden shared by seller and buyer.
* Same deadweight loss.
* The share of tax burden only depends on the demand and supply elasticity, i.e. whoever is more elastic bears less burden.
  · Share of tax paid by consumer $= \frac{E_s}{E_s+|E_d|}$
  · Cigarrets tax borne mostly by smokers due to high demand elasticity.
  · Payroll tax mostly born by workers (supplier of the labor) due to low supply elasticity.

• Numerical example:

$$Q_d = 100 - 6P$$

$$Q_s = 50 + 4P$$

- Equilibrium with no tax:

$$Q_d = Q_s \rightarrow P^* = 5, Q^* = 70$$

- A tax rate of \$1 is introduced.
- Now you need to differentiate P_b and P_s in the demand and supply equation.

$$Q_d = 100 - 6P_b$$

$$Q_s = 50 + 4P_s$$

- Tax introduces a wedge between P_b and P_s. Always rember $P_b > P_s$ after tax.

$$P_b = P_s + \underbrace{1}_{\text{the wedge equal to tax}}$$

- Equilibrium after tax means

$$Q_s = Q_d$$

- Solve the equation. New quantity

$$Q' = 67.6 < Q^*$$

- Prices

$$\rightarrow P_s = 4.4 < P^*, P_b = 5.4 > P^*$$

- DWL:

$$\frac{1}{2}(Q^* - Q') \times \text{tax} = 2.4$$

- tax revenue

$$Q' \times \text{tax} = 67.6$$

- Share of tax burden.

$$\text{tax paid by consumer} = \frac{E_s}{E_s + |E_d|}$$

. Rember that E_s and E_d shall be computed at the Q' and P_s and P_b, respectively.

1.2 Subsidy

- What's the difference between tax and subsidy?
  - Tax is the government **collecting money** from the market while subsidy is government **paying money** to the market participants.
  - Tax leads to **lower quantity** (than market equilibrium) sold, while subsidy leads to **higher quantity**.
  - Tax makes buyer pay **more** than what producers receive. Subsidy make the buyer pay **less** than what producer receive.
  - Tax brings a revenue to the government. Subsidy brings spending to the government.
- What's the same between the two?
  - Both induce inefficient outcomes than the market equilibrium, i.e. positive deadweight loss.
    * The inefficiency from tax is there is **not enough trade**, i.e. some consumers would have bought the goods without the tax, while the one from a subsidy is **too much trade**, many producers should have not produced the good without the subsidy.
  - Remember that when we talk about social surplus with tax/subsidy, we are implicitly talking about consumer + producer + government.
  - CS and PS increase after subsidy but CS+PS- Government Spending <0.

Session 5. Consumer's behaviors

Tao Wang

October 8, 2020

1 Preference

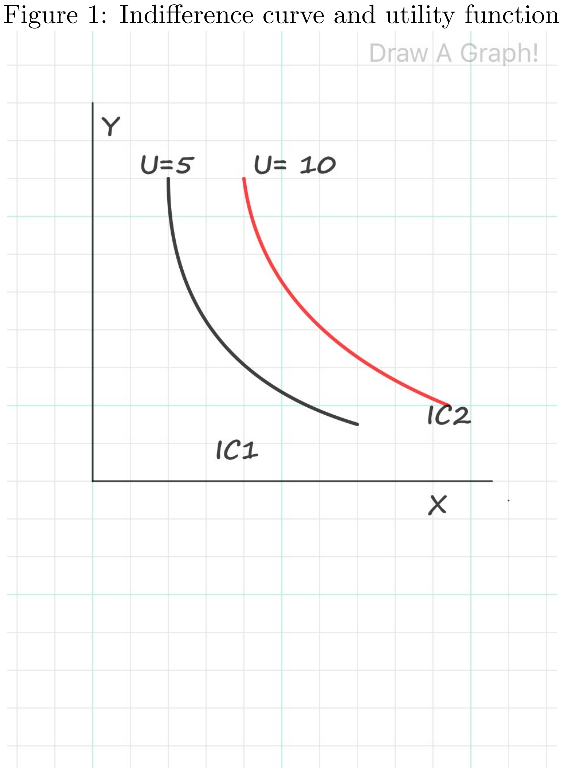
- Utility function: $U(X, Y)$ is the utility from consuming a bundle of X and Y
  - Why utility function?
    - We want to not only rank choices(**ordinal**) but also compare them (**cardinal**). We may easily know that having 1 apple and 2 bananas is better than 1 apple and 1 banana but by how much?
    - **Inter-personal comparison and social welfare aggregation**. I love apples more than banana and you love banana more than apples. Who is happier with 1 apple and 1 banana? How much happier the society will be if I exchange 1 apple for your 1 banana?
    - **Inter-temporal comparison**. Are 1 apple and 1 banana as satisfactory in tomorrow as today?
  - Marginal utility
    - Definition: the **additional** utility from consuming **one more unit** of the good.
    - Intuition: the extra satisfaction by having the next bottle of water when you are desperately thirsty when hiking (high MU) versus when you are sitting comfortably in a hotel room with a big pitcher of water in front of you (low MU).
    - Math:
$$MU_X = \frac{\delta U(X, Y)}{\delta X} = U'_X(X, Y)$$
      - Remeber that it is the partial derivative. Y is a constant when you take derivative with respect to X.
    - Graph: the slope of $U(X, Y)$ with respect to X at a **given** Y.
    - Examples
      - $U(X, Y) = X + 3Y$: $MU_X = 1$, $MU_Y = 3$, i.e. MU of each good is not dependent on the other.

- $U(X,Y) = XY$: $MU_X = Y$, $MU_Y = X$, i.e. MU of each good is dependent on the other.
- $U(X,Y) = X$: $MU_X = 1$, $MU_Y = 0$, i.e. Y does not affect the utility, or the utility from Y is a constant.
- $U(X,Y) = 1$: $MU_X = 0$, $MU_Y = 0$, i.e. neither X and Y affects the utility.
- $U(X,Y) = lnX + Y^2$: $MU_X = \frac{1}{X}$, $MU_Y = 2Y$, i.e. MU of X decrease with X and MU of Y increase with Y.

– Relation between preference, the utility function, and the indifference curve

  * A particular indifference curve represents all combinations of X and Y that give one the same level of utility. See Figure 1.

Figure 1: Indifference curve and utility function

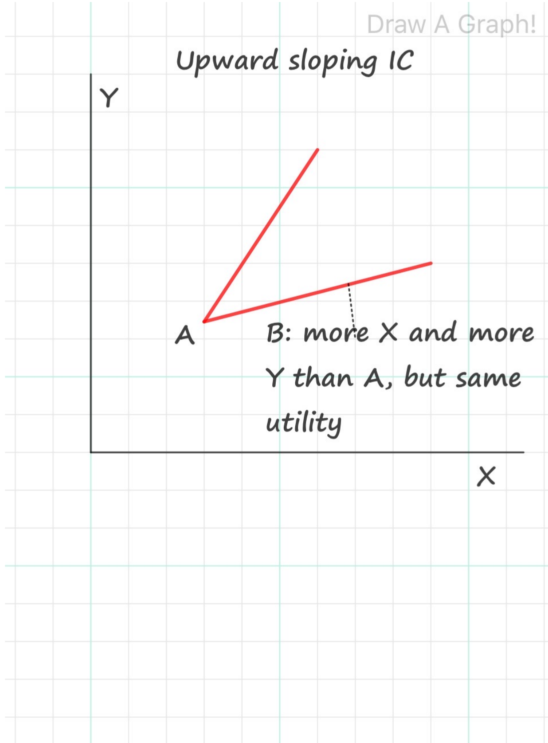


  * **Completeness and rankability:** the consumer can always compare different bundles and decide if he/she prefers one to the other. $\leftrightarrow$ Each combination of X and Y needs to to be on a particular IC. We can always draw ICs for any bundle. $\leftrightarrow$ $U(X,Y)$ is defined over all positive values of X and Y. For in-

stance, $U(X,Y) = log(-X) + log(-Y)$ is not defined for $X > 0$ and $Y > 0$.

* **More is better**: IC downward sloping $\leftrightarrow$ If I give up some X, I have to get more of Y so that I am equally happy. $\leftrightarrow$ utility function increase in X and Y. For instance, $U(X,Y) = -XY$ decreases with X and Y. Or see Figure 2 for a counter-example.

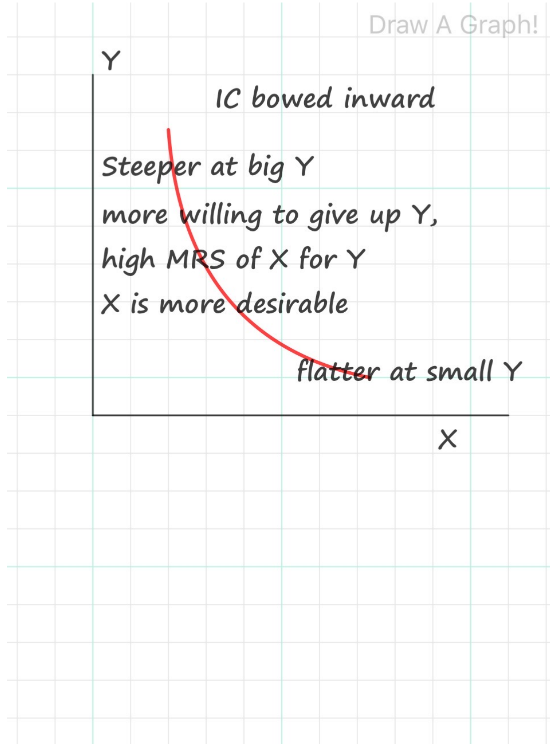
Figure 2: An upward indifference curve: more is not better



* **Diminishing marginal utility:** One is more willing to give up the abundant things he/she has to exchange for the things he/she has little. $\leftrightarrow$ IC is bowed inward $\leftrightarrow$ the IC is steeper for a bigger Y and flatter for a smaller Y. $\leftrightarrow$ $MRS_{X,Y}$ decreases in X and increase in Y. See Figure 3.
* **Transitivity**: If you prefer A to B and B to C, then you prefer A to C. $\leftrightarrow$ two ICs never cross. See Figure 4. .

- Marginal rate of substitution (MRS)
  - Definition: MRS of X for Y is the amount of Y a consumer is willing to give up in exchange for an additional unit of X **so that he/she**

Figure 3: Standard indifference curve that is bowed inward



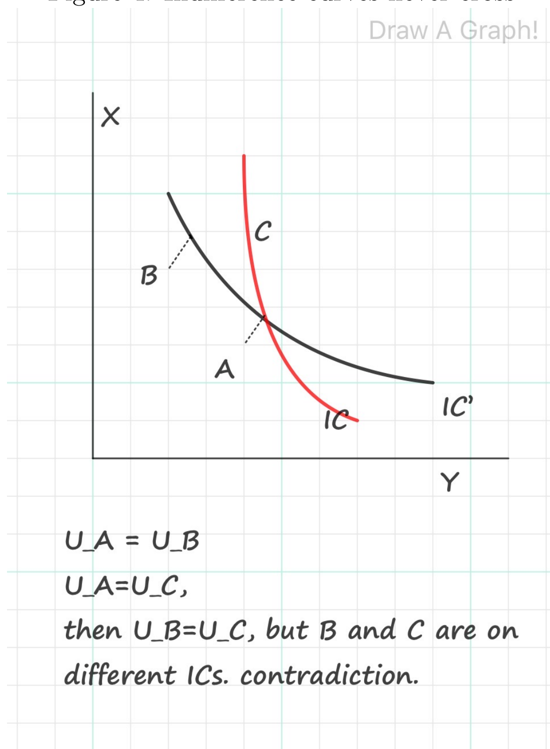
is equally happy, or stay in the same level of utility.

- Intuition: you want to measure the relative desire you have for X versus Y. For instance, if you desire X, you are willing to give up a lot of Y in exchange for it, a high MRS of X and Y. [1]
- Math: $MRS_{X,Y} = -\frac{\Delta Y}{\Delta X}$. IC has a negative slope. Adding a negative sign makes the MRS positive, as a convention.
- Graph: the slope of the IC **at a particular X and Y**. Steeper ICs (Y being vertical) mean high MRS of X for Y.
- Relationship between MRS and marginal utility. $MRS_{X,Y} = \frac{MU_X}{MU_Y}$.
  * Proof: in indifference curve, we know that the utility change from change in X will be exactly equal to the change in utility from change in Y, so

$$MU_X \delta X + MU_Y \delta Y = 0$$

[1] The easiest way to remember this is that "X goes first then Y, so X is more desirable' for higher MRS of x for Y'.

Figure 4: Indifference curves never cross



Therefore,

$$\text{Slope of IC} = \frac{\delta Y}{\delta X} = -\frac{MU_X}{MU_Y} = -MRS_{X,Y}$$

 * Intuition: the relative desire of X versus Y is higher if MU of X is higher relative to that of Y.

 - Why do we care about it? We want to compare this relative desire for X and Y with the relative cost/price of X and Y to determine the optimal allocation between X and Y.

2 Budget Constraint

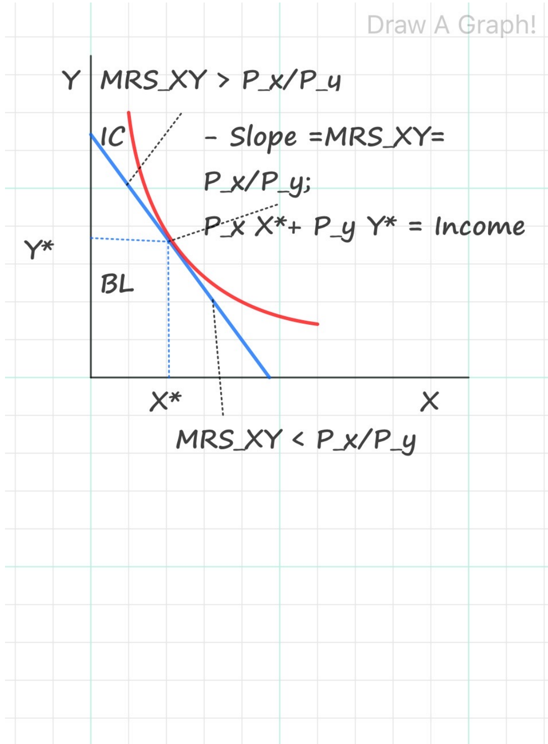
- Definitition:the combinations of X and Y that a **given level** of income could afford.
- Math: $Income = P_Y X + P_Y Y$ for a given income/budget.
- Graph: budget line

- $|Slope| = \frac{P_X}{P_Y}$, when Y is on the vertical axis.
- Intercept: vertical intercept $Income/P_Y$ and horizontal intercept: $Income/P_X$.
- Higher income leads to parallel shift of BL outward
- Higher price of Y (vertical good) versus horizontal leads to flatter BL.

3 Optimization

- General condition for optimal consumption.
  - Verbal: the utility is maximized when (1) the relative desire of X and Y is equal to the relative cost of buying them; (2) all the income is spent.
  - Intuition: (1) if spending one more \$1 on X brings higher marginal utility than on buying Y, one should buy more X and buy less Y. (2) If there is income left; one can always use it to buy more of X or Y to increase utility.
  - Math:
  $$MRS_{X^*,Y^*} = \frac{MU_X(X^*, Y^*)}{MU_Y(X^*, Y^*)} = \frac{P_X}{P_Y}$$
  and
  $$P_X X^* + P_Y Y^* = Income$$
  - Graph: the IC is tangent with the budget line at the optimal consumption. See Figure 5.
- Special cases under non-standard preference/ICs or budget constraints In many cases, we cannot apply the principle $MRS_{X,Y} = \frac{P_X}{P_Y}$ to find the optimal consumption.
  - **Perfect substitutes**: both the slopes of IC and budet line are constant. Spend all money on X if X is more desirable than Y given the relative cost. An example of utility function is $U(X, Y) = X + Y$. See Figure 6.
  - **Perfect complements**: $U(X, Y) = min(X, 2Y)$. Then optimal consumption is $X^* = 2Y^*$. See Figure 7.
  - **Bad things/negative utility**: $U(X, Y) = -XY$, any positive X and Y give you lower utility. So optimal consumption requires X=Y=0, i.e. not consuming X and Y at all.
  - **Something you do not care**: for instance, you do not care about X at all, i.e. there is no X in the utility function, $U(X, Y) = 3Y$. The IC becomes a horizontal line. Optimal consumption is to spend all money on Y and 0 on X. See Figure 8.

Figure 5: Optimal consumption under standard IC and budget constraint



- **Quantity limits**: the budget is kinked now and is flat for any quantity higher than the quantity limit. See Figure 9.
- **Quantity discounts**: budget line has a kink now, i.e. the slope of the budget/price depends on the quantity.

Figure 6: Optimal consumption for perfect substitutes

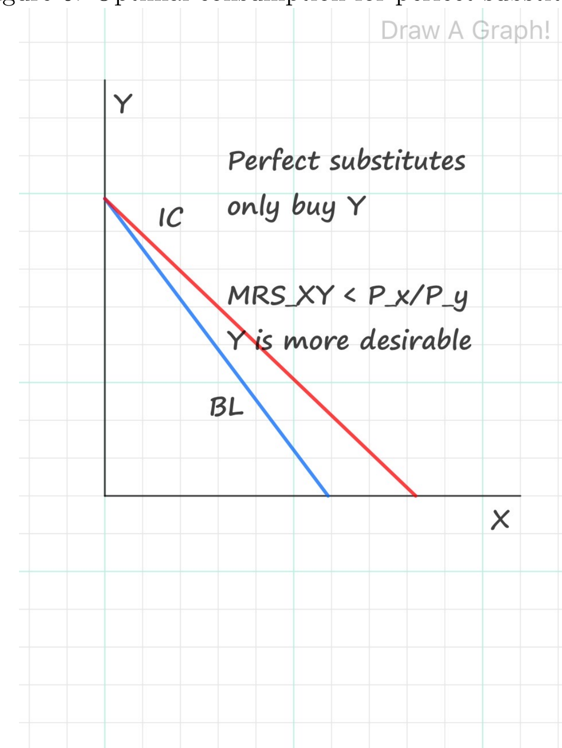


Figure 7: Optimal consumption for perfect complements

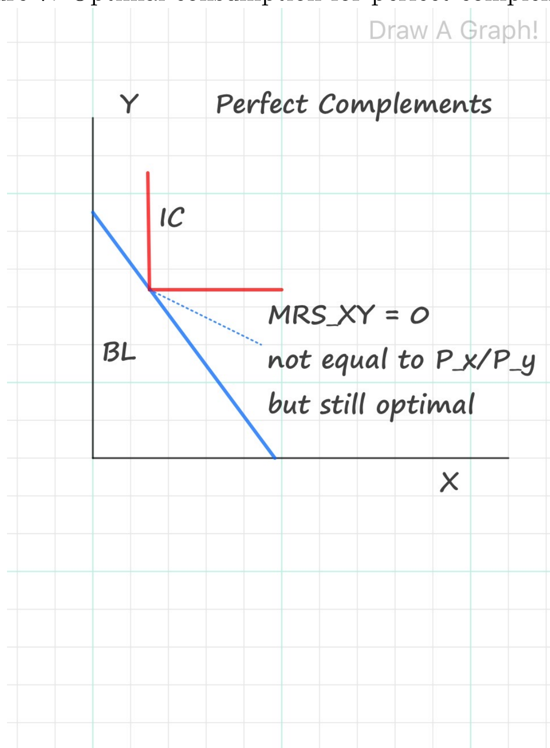


Figure 8: Optimal consumption if you do not care about one good

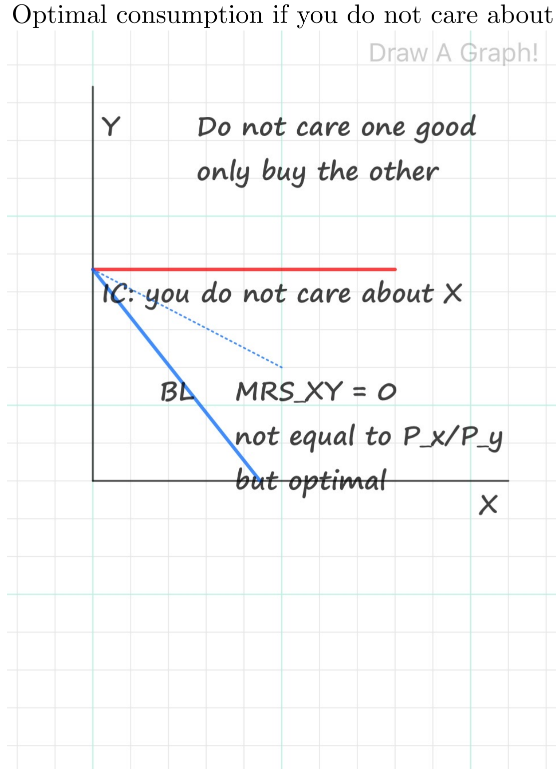
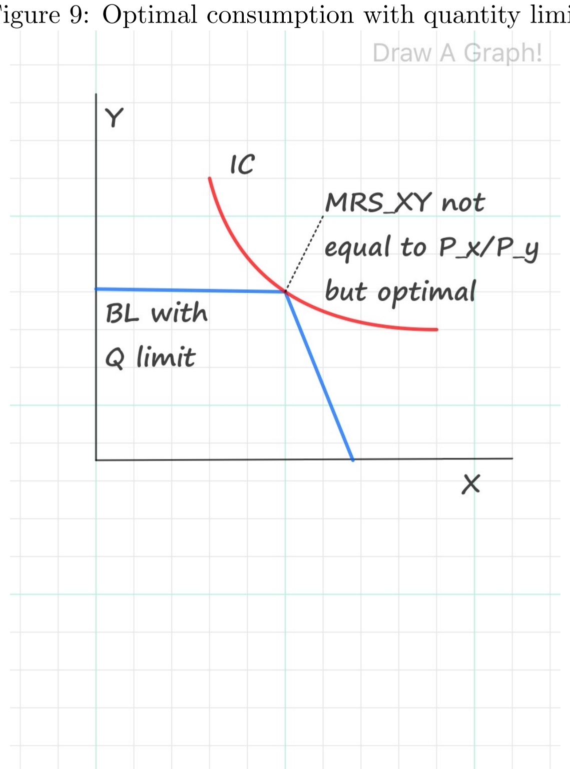


Figure 9: Optimal consumption with quantity limit



Session 6. Demand

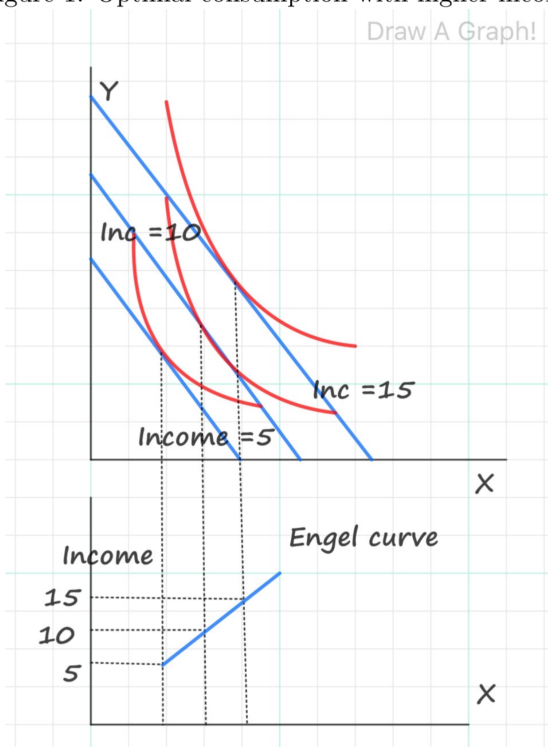
Tao Wang

October 9, 2020

1 Demand responses to income change

- Graph: parallel shift outward (inward) of the budget line for higher income (lower income). Find where the IC is tangent with the new budget line.

Figure 1: Optimal consumption with higher income



- Math:

- Example 1. $U(X,Y) = XY$, and $P_X = 1$ and $P_Y = 4$. Imagine that the income increases from 10 to 20 and price stays the same. Plot the Engel curve for X as a function of income.
- Anwer: No matter what the income is, in the optimal consumption, we always have.

$$MRS_{X,Y} = \frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}$$
$$\frac{Y}{X} = \frac{P_X}{P_Y} = 1/4$$

To put it differently, the ratio of X and Y for a given income is the same.

When income is 10, this means that

$$P_X X^* + P_Y Y^* = \mathbf{10} \rightarrow X^* = 4Y^* = 5$$

When income is 20, this means that

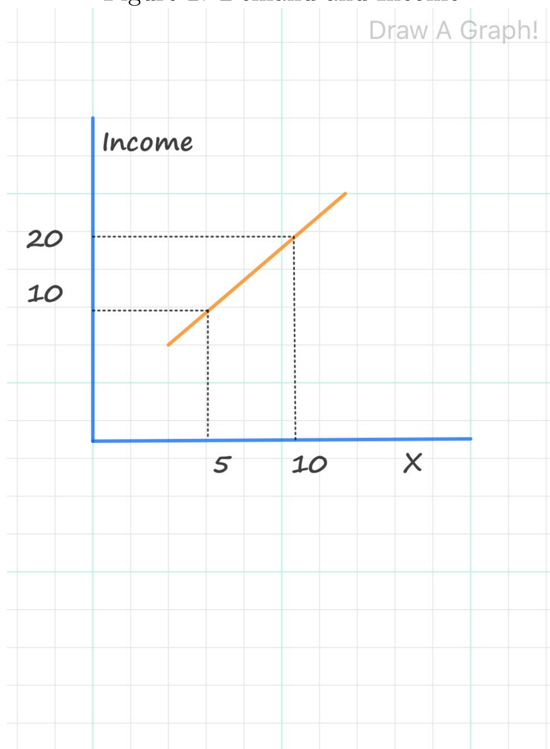
$$P_X X' + P_Y Y' = \mathbf{20} \rightarrow X' = 4Y' = 10$$

We now have the demand for X at the income of 10 and 20 and we can plot an Engle curve based on the two points. See Figure 2.

- Question: what determines the shape of the Engel curve?
  - Preference, the utility function, or the shape of your indifference curve.
  - **upward sloping**: higher income leads to higher demand, normal good, i.e. going to the restaurants.
  - **linear**: i.e. you want to double your demand if you double your income.
  - If it gets more flattened as income increases, proportional change in income leads to a smaller proportional change in the demand.
  - **downward sloping**: higher income leads to lower demand. For instance, inferior good, i.e. junk food, staple.
  - It could be upward sloping first and then downward sloping. For instance, you take more buses as income rise and then decrease when you get rich enough.

- Income elasticity: the percentage change in the demand for a good associated with one percent increase in income.
  - It is not equal to the slope of the Engel curve. But they have the same signs.
  - Income elasticity of different types of good
    - Normal good: income elasticity is positive.
    - Inferior good; negative

Figure 2: Demand and Income



* $*$ Luxury good: positive and greater than 1. This is why higher-income families have a bigger share of luxury good spending than low-income families.
* $*$ Necessity good: positive and smaller than 1.

- Math: in the example above, the income elasticity for X at the income of 10[1] is

$$E_X^I \text{at income of } 10 = \frac{\Delta X}{\Delta \text{Income}} \frac{Income}{X^*} = \frac{10-5}{20-10} \frac{10}{5} = 1$$

$$E_X^I \text{at income of } 20 = \frac{\Delta X}{\Delta \text{Income}} \frac{Income}{X'} = \frac{5-10}{10-20} \frac{20}{10} = 1$$

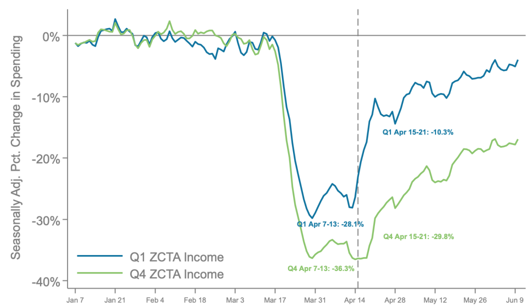
Notice that the income elasticity are constant at different income. This is a special case. We may not always have this.

• Interesting things about the Engel curve.

  - Consumption during the pandemic. Why did high-income households' consumption drop more than low-income households?

[1] Always remember, income elasticity corresponds to a particular level of income

Figure 3: Consumption during pandemic



2 Demand responses to price change

- Graph: a budget line rotates around the interception of the good axis whose price remains the same.
  - Flatter if the horizontal good is more expensive.
  - Each new budget line corresponding to a price has its optimal consumption.
  - We can plot the demand curve by collecting all of these points. See Figure 4.
- Math:
  - Same example as above. Fixing the income at 10 and increases price of Y from 4 to 5.
  - Answer: following price change, the optimal condition changes.

    $$\frac{Y'}{X'} = \frac{P'_X}{P'_Y} = 1/5$$

    Combining with

    $$P_X X' + P_Y Y' = \mathbf{10}$$

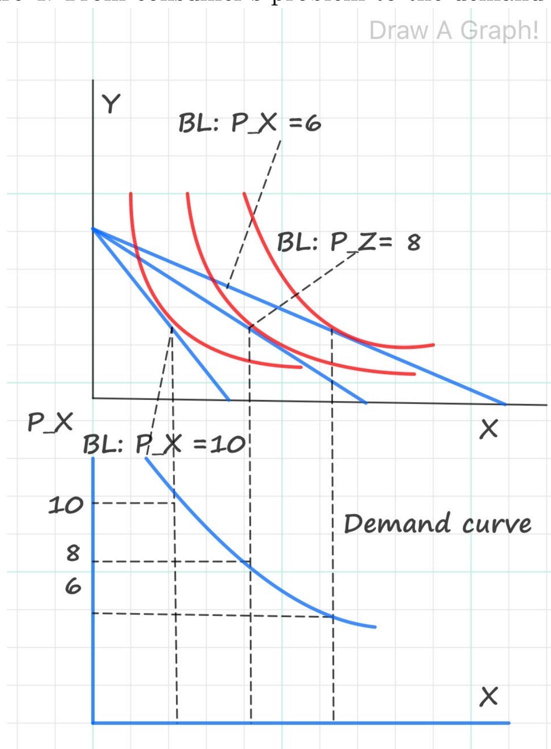
    We obtain

    $$X' = 5$$

    $$Y' = 1$$

    Notice the demand for X is the same as before and demand for Y is higher.
  - Intuition:when Y is more expensive, you reduce demand for Y. But why does the demand for X stay the same? The key is to differentiate substitution and income effects.
- Substitution and income effect. See Figure 2.

Figure 4: From consumer’s problem to the demand curve



– Now let us imagine Y is suddenly more expensive relative to X. What would this mean for the optimal demand of the consumer for X and Y?

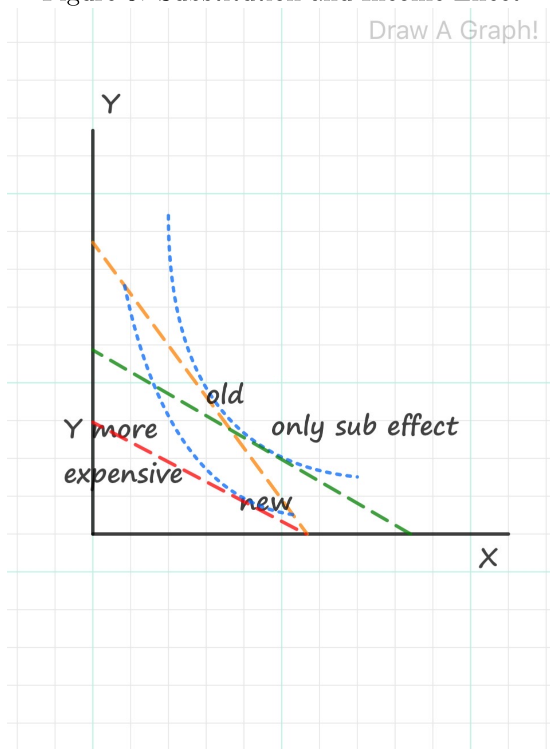
– First, we may expect, not surprisingly, the consumer demands less Y relative to X. This is straightforward. We called the substitution effect.

– The second effect is more subtle. Since the price of Y is more expensive, the *real* income of the consumer measured by the price of Y is lower. This will make her feel poorer and thus willing to buy less of both X and Y. This is called the income effect.

– Graphically, how do we show the two effects separately? (See Figure 2.)

  ∗ Starting from the old point, move along the same IC to find the point in which it is tangent with a budget line under the new price ratio. Since at this point, she attains the same level of utility at a different price. The move from old optimum to this point only comes from substitution effect, i.e. given the new

Figure 5: Substitution and Income Effect



price, how would substitute between two goods to stay as equally happy.

* From the new point only from the substitution effect, move the budget line inward to equal the same level of total income. Then the new tangent point with the IC at a lower level of utility is the new optimum resulting from both sub and income effect.

- Type of goods
  - All good: substitution effect of **a price increase of itself** (another substitute good) is negaitive (positive) because you substitutes away the good as it is more expensive.
  - Normal good: income effect of **a price increase of itself** (or another good) is negative because the higher price lowers your real income and demand less of the good.
  - Inferior good: positive.
  - Giffen good: income effect of **a price increase increase** is so positive that it dominates the substitution effect. So that the demand increases with the price.

3 Demand responses to the price change of other goods

- Shift outward of the demand curve if the price of substitutes is higher
- Shift inward of the demand curve if the price of complements is higher

Session 7. Producers' Behaviors: Practice Questions

Tao Wang

October 15, 2020

- Basic assumptions about production

Figure 1: Assumptions about the production

1. For parts (a) to (d), determine which basic assumption of production is violated.

   a. The local coffee shop serves 400 customers per day with 8 employees, 420 with 9 employees, or 450 with 10 employees.

   b. The French government makes it very difficult for Antoine, a bakery owner, to hire new workers or fire existing workers. On the other hand, Antoine can instantly change the quantity of the ovens and mixers in his shop.

   c. Business is booming at Jordan's custom bicycle factory. When she increases the number of employees from 40 to 45, her production falls from 500 bikes per month to 480.

   d. Smartphone apps are all the rage . . . but finding skilled programmers is becoming more difficult. The more programmers Google (a large employer) hires, the more it must pay each programmer.

  - Answer:
    - a. It violates the law of diminishing return to labor, or decreasing marginal product of labor. From 8 employees to 9: $\Delta Y = 20$; From 9 employees to 10: $\Delta Y = 30 > 20$.
    - b. Firms can buy as many labor inputs as they want at the fixed price. But here the government regulation makes the firm unable to do this.
    - c. More inputs bring more output, or marginal product of labor is positive.
    - d. Firms are price takers/perfectly competitive. But here Google's hiring affects the price of the labor.

- Return to scale
  - General rule of thumb: any production function form $Q = AK^aL^b$ is
    CRS if $a + b = 1$;
    DRS if $a + b < 1$
    IRS if $a + b > 1$.

Figure 2: Production functions and return to scale

19. Determine whether each of the production functions below displays constant, increasing, or decreasing returns to scale:

a. $Q = 10K^{0.75}L^{0.25}$

b. $Q = (K^{0.75}L^{0.25})^2$

c. $Q = K^{0.75}L^{0.75}$

d. $Q = K^{0.25}L^{0.25}$

e. $Q = K + L + KL$

f. $Q = 2K^2 + 3L^2$

g. $Q = KL$

h. $Q = \min(3K, 2L)$

- Answer:
  - a. CRS.
  - b. IRS. Double both inputs:

$$Q' = [(2K)^{0.75}(2L)^{0.25}]^2 = (2^{0.75+0.25})^2 Q = 4Q > 2Q$$

  - c. IRS.
  - d. DRS.
  - e. IRS. Double inputs. $Q' = 2K + 2L + 4KL > 2Q$
  - f. IRS.
  - g. IRS
  - h. CRS.

- Short-run problem. Figure 3.
  - Answer
    - a. In short run, capital is fixed at $\bar{K} = 9$. So production function is

$$Q^{SR} = 100\sqrt{\bar{K}L} = 300\sqrt{L}$$

    which is only a function of L.
    - b. $Q^{SR}(L = 1) = 300$
    $Q^{SR}(L = 2) = 300\sqrt{2}$
    $Q^{SR}(L = 3) = 300\sqrt{3}$
    $Q^{SR}(L = 4) = 600$
    $Q^{SR}(L = 5) = 300\sqrt{5}$
    - c. $MP_L = \frac{dQ^{SR}}{dL} = \frac{150}{\sqrt{L}}$, decreasing in L, implying the diminishing return to labor.

Figure 3: Producer's problem in the short-run

4. Nobody fixes more fender benders than Crazy Bob! At his auto body shop, the production function showing the number of cars repaired each year is $Q = 100\sqrt{KL}$, where *K* is the number of arc welding machines available and *L* is the number of employees. Currently, *K* is fixed at 9.

a. Write an equation for Crazy Bob's short-run production function, showing output as a function of labor only.

b. Calculate the total number of cars Bob can repair each year for *L* = 1, 2, 3, 4, and 5.

c. Calculate the marginal product of labor for each of Bob's first five workers. Does the MP_L diminish?

d. Calculate the average product of labor for Bob's first five workers. Is the MP_L greater than, equal to, or less than the AP_L at each level of employment? Why?

$$MP_L(L=1) = 300 - 0 = 300$$
$$MP_L(L=2) = 300(\sqrt{2}-1)$$
$$MP_L(L=3) = 300(\sqrt{3}-\sqrt{2})$$
$$MP_L(L=4) = 300(2-\sqrt{3})$$
$$MP_L(L=5) = 300(\sqrt{5}-2)$$

* d.
$AP_L = \frac{Q^{SR}}{L} = \frac{300}{\sqrt{L}}$, decrease with L.
$$AP_L(L=1) = 300 - 0 = 300$$
$$AP_L(L=2) = 150(\sqrt{2}-1)$$
$$AP_L(L=3) = 100(\sqrt{3}-\sqrt{2})$$
$$AP_L(L=4) = 75(2-\sqrt{3})$$
$$AP_L(L=5) = 60(\sqrt{5}-2)$$

- Marginal rate of technological substitution (MRTS).
  - Question: suppose the production function is $Q(K,L) = ln(K) + 2L$. Solve following questions
    * a. Compute the MRTS of capital for labor for four scenaiors (1) K=10 and L=5; (2) K =5 and L=10; (3) K=5 and L=0; (4) K=0 and L=5.
    * b. Sketch (qualitatively) an isoquant curve for this production function based on the four sets of values computed above.
    * c. What does a higher MRTS of K for L imply for the nature of production?
    * d. Imagine the production function is instead $Q(K,L) = K + 2L$, what is MRTS? Why is it a constant? What shape of isoquant curve?
  - Answer

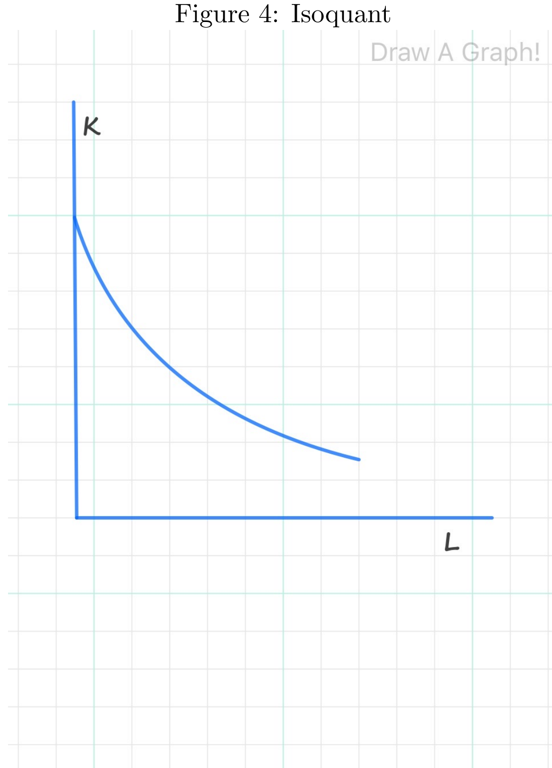
* a.

$$MRTS_{K,L} = \frac{MP_K}{MP_L} = \frac{1}{2K}$$

(1) $MRTS = 1/20$;
(2) $MRTS = 1/10$;
(3) $MRTS = 1/5$;
(4) $MRTS = +\infty$.
The marginal product of labor is a constant and the marginal product of capital decreases with K. Therefore $MRTS$ of K for L decreases with K.

* b. See Figure 4.
Two important things. First, the isoquant intersects with K but not with L (and will never). L can be perfectly substituted for K but not the opposite.
Second, the isoquant is steeper as K is abundent (low $MRTS_{K,L}$) and flatter as K is little, (high $MRTS_{K,L}$)

Figure 4: Isoquant



* c. Higher $MRTS$ of capital for labor implies that capital can bring higher marginal product; more labor can be substituted

for a little capital to produce the same amount.

  * d. $MRTS_{K,L} = 1/2$, a constant. It is constant because the marginal product of both K or L is constant. Therefore isoquant is a linear line.

- Marginal product

Figure 5: Marginal product

7. Suppose that a firm's production function is given by $Q = K^{0.33}L^{0.67}$, where $MP_K = 0.33K^{-0.67}L^{0.67}$ and $MP_L = 0.67K^{0.33}L^{-0.33}$.

a. As L increases, what happens to the marginal product of labor?

b. As K increases, what happens to the marginal product of labor?

c. Why would the MP_L change as K changes?

d. What happens to the marginal product of capital as K increases? As L increases?

  - Answer
    * a. $MP_L = 0.67K^{0.33}L^{-0.33}$ decreases with L.
    * b. MP_L increases with K, more capital makes labor more productive marginally.
    * c. How many more pizzas you can make by having one more worker also depends on how many pizza machine you have.
    * d. MP_K decreases with K and increases with L.

Session 8. Cost Curves and Cost Function

Tao Wang

October 29, 2020

- The cost we "care" and we do not "care"
  - **Opportunity cost**: we always care about it
  - **Sunk cost**: we do not care. But there might be reasons for which we need to care
    - We do not care because it has been incurred and and is unrecoverable. It cannot be undone. [1]
    - Some people call it **long-run fixed cost**.
    - Example 1: doctors' expense going to medical school is the sunk cost.
    - Example 2: home owner's expenses paid out on the new house is the sunk cost. Or you can say the difference between that spending and the current market value as the sunk cost. Neither of them should affect your decisions. It is how much you can sell it that matters.
    - Example 3. you spent \$50 purchasing a stock last month and now it is only worth of \$20. The sunk cost is \$30. What matters for the current investment decision is only that \$20. If you think the price is going to be \$30, hold the stock. If it is going to be \$10, sell it before you lose more.
    - Although rational forward-looking decisions should not be influenced by sunk cost, we may have other rationale to care about what is seemingly sunk cost: reputation concerns, commitment device, information, etc.
      - Not leaving the theatre in the middle of the boring performance may not be due to sunk-cost fallacy. Instead, simply because you do not want to disturb the audience around you.
- Why do we need cost functions/curves?
  - Cost minimization (for a given quantity) is needed for profit maximization. To maximize profits, the firm takes two steps.

[1] A life lesson here: look forward and forget about the past results you cannot change.

    * First, how much I want to produce given the cost and price of the good?

$$\max_{Q} \quad Q \times P - Q \times \mathbf{ATC}$$

$$or \quad \max_{Q} \quad Q \times P - \mathbf{TC}$$

The first order condition

$$\frac{\Delta \text{TC}}{\Delta Q} = \mathbf{MC} = P$$

$$\rightarrow Q^*$$

    * Second, given that target quantity Q^*, what is the cost-minimizing way of producing it?

$$\min_{\{L,K\}} LW + RK$$

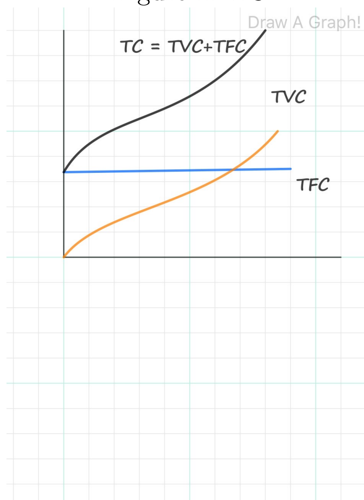
$$s.t. \quad F(L,K) = Q^*$$

$$\rightarrow L^*, K^*$$

  - Cost functions characterize the shapes of ATC, TC, and MC, in both the short-run and long-run by focusing on the second step of the question above.
  - Once we figure out them, the first step of the question above is easy.

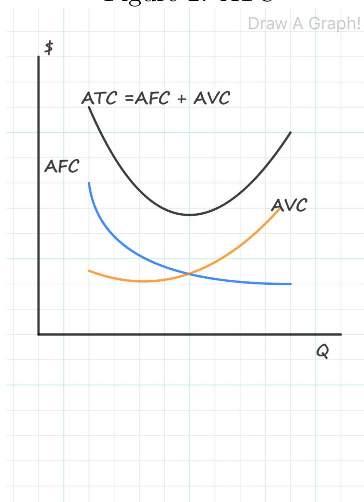
- Different costs in the short run
  - Fixed cost and variable cost
    * TC = TFC+TVC . See Figure 1.
    * ATC = AFC + AVC See Figure 2.
      · AFC is monotonically decreasing with Q
      · AVC first decreases and then increases, i.e. more workers improves efficiency first and then decreases it.
    * $MC = \frac{\Delta TC}{\Delta Q}$, the slope of TC curve. See Figure 3.
      · MC increases after some point due to the law of diminishing return.
      · It may decrease in the beginning because of increasing return.
    * ATC and MC. See Figure 4.
      · MC always pulls ATC toward it.
      · ATC and MC intersect at the minimum of the ATC.
      · This is the same reason why average APL intersects with MPL at its maximum.
  - Short-run and long-run

Figure 1: TC



- ∗ The key difference between SR and LR and why it matters for cost functions
  - · SR: capital is fixed and only labor is adjusted. LR: both can be adjusted.
  - · SR: **No** cost minimization is involved!. LR: cost is minimized.
  - · SR: you stay in one **given** SRAC or SRMC; LR: you can **choose** which SRAC or SRMC you want to be.
  - · This means that for any given quantity, the quantity of the capital fixed in SR may not be necessarily at the cost-minimizing point. But it is so in the LR.
- ∗ TC in SR and LR. See Figure 5.
  - · TC curves at different fixed capital in the SR are above LR, i.e. more costly to produce pizzas when you can only change the number of workers but cannot change the size of the kitchen.
  - · The only intersection between the two is where the fixed capital minimizes the cost.
  - · Intuition: you do better in minimizing the cost if you have control over two inputs instead of just one.
- ∗ ATC in SR and LR. See Figure 6.
  - · LR ATC curve is an "envelope" of all SR ATC curves.
  - · Intuition: imagine you have three sizes of the factory you could choose. Each size of the factory is represented by an SR ATC curve. For each quantity, you want to produce, choose

Figure 2: ATC



among all three SR ATC curves the one that produces that quantity at the minimum cost. You end up getting the cost-minimizing curve (purple in Figure 6). Now, imagine instead there is an infinite number of sizes of factors you could choose from. then the purple curve converges the LR ATC curve.

·

∗ MC in SR and LR. See Figure 7.

- LR MC intersects with every SR MC for a particular fixed level of capital at the quantity **at which SR ATC conincide with LR ATC**.
- Or LR MC intersects with every SR MC for a particular fixed level of capital at the quantity **where the SR cost is minimized at that level of capital**.
- LR ATC also intersects with LR MC at its minimum, the same as in the SR.

Figure 3: MC

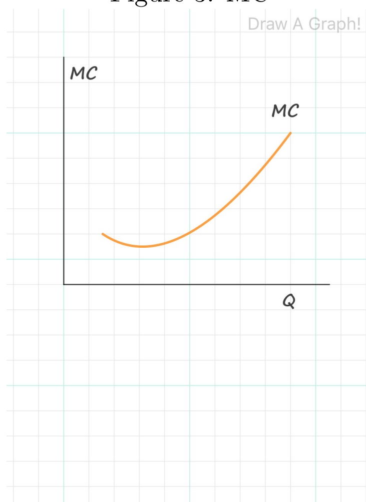


Figure 4: ATC and MC

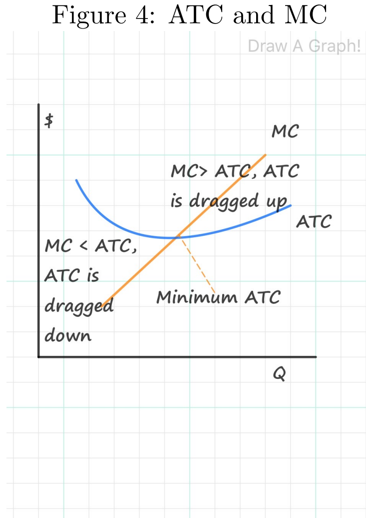


Figure 5: TC in the SR and LR

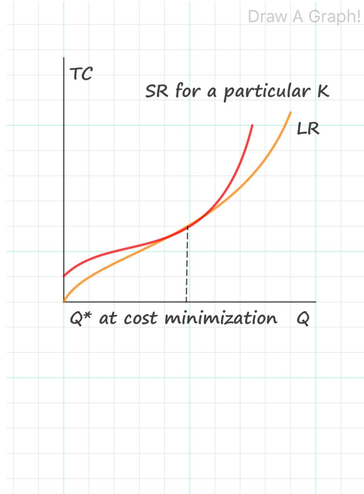


Figure 6: ATC in the SR and LR

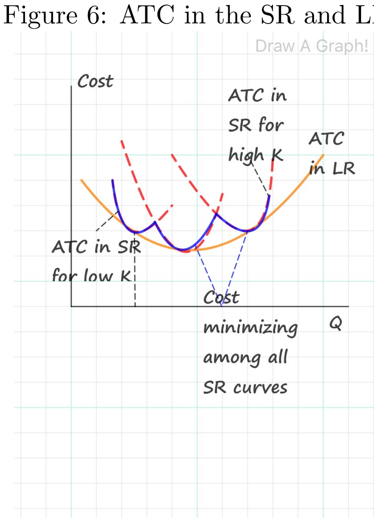
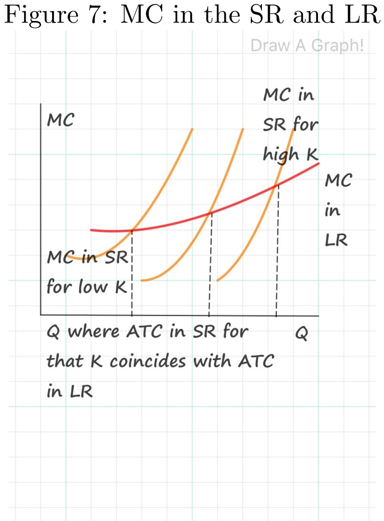


Figure 7: MC in the SR and LR



Session 9. Firms under Perfect Competition

Tao Wang

November 5, 2020

1 The key characteristics of perfect competition

- A large number of firms, i.e. in theory, an infinite number of firms.
- Produce homogeneous goods. Differentiable goods give monopoly power instead.
- Free entry/exit, i.e. no barrier either from government regulations or from the market.
- Firms are both price takers, i.e. taking the price of the good and the price of inputs as given.

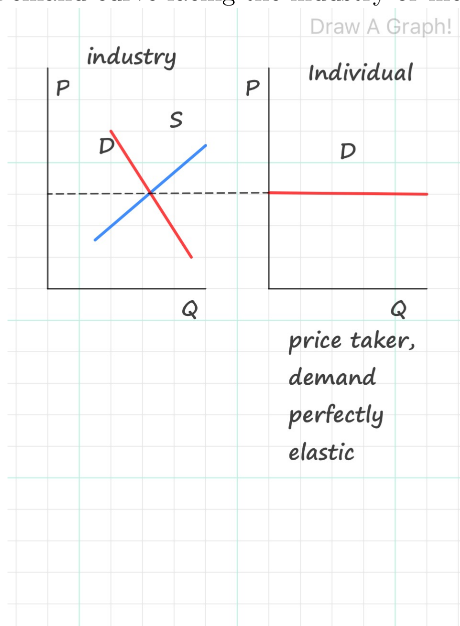
2 Demand curve and firm's profits maximization in perfect competition

- Industry demand curve is **downward sloping**. See left figure in Figure 1.
- Individual demand curve is **horizontal**. See right in Figure 1.
  - Perfectly elastic, i.e. the consumer easily shifts to another firm if one firm increases the price
  - The price of the good is taken as given.
  - Firm's decision of how much to produce does not affect the price of the good.
  - In the firm's profits maximization problem, the price is not a function of Q.

    To see why it is important, think about the profit maximization problem below.

$$max \quad \underbrace{QP}_{TotalRevenue} - \underbrace{TC}_{TotalCost}$$

Figure 1: Demand curve facing the industry or individual firm



Take derivative with respect to Q as the following

$$\underbrace{MR}_{MarginalRevenue} = \underbrace{MC}_{MarginalCost}$$

When P is not a function of Q(which is not the case in monology),

$$MR = P$$

.

Therefore, the optimal condition for a firm's profit maximization is

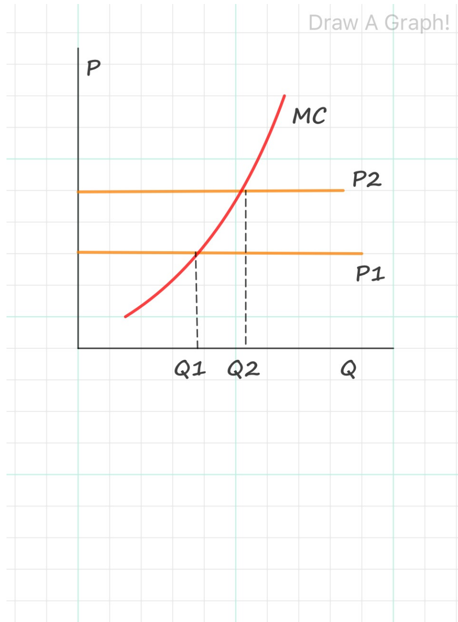
$$P = MC$$

3 Supply curve

- Individual supply curve in short-run
  - How to derive a supply curve? For a given P, the firm picks the Q_1 such that $P_1 = MC(Q_1)$ [1]. For another P_2, the firm picks the Q_2 such that $P_2 = MC(Q_2)$. Connecting these two points, you get the short-run supply curve. See Figure 2.

[1] MC(Q) represents MC as a function of Q.

Figure 2: Profits Maximization under different P

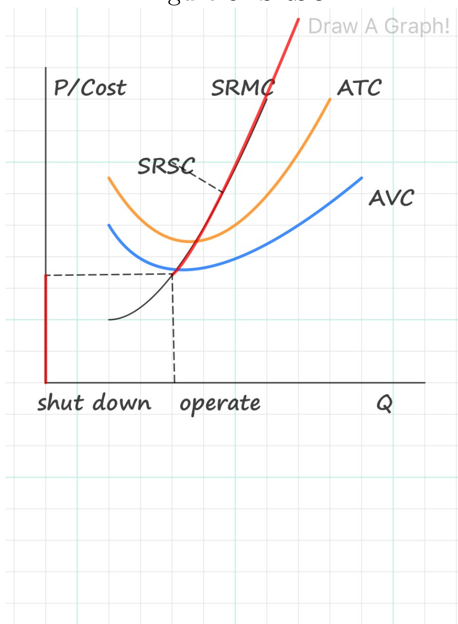


  - This turns out to be exactly the marginal cost curve in the short run.
  - But the part of the marginal cost curve for which the price(or MC) is below average variable cost should be excluded. This is because the firm will decide to shut down to avoid loss if the total variable cost is higher than the total revenue. See Figure 3.
  - Profits are the difference between total revenue and the total cost. See Figure 4

- Individual and industry curve
  - Different firms all have upward supply curves with different cost curves.
  - Horizontally sum them all up gives you industry short-run supply curve.
  - The industry short-run supply curve is upward sloping because
    * Individual supply curve is upward sloping
    * Firms with higher costs start producing goods at a higher price.

- Producer profits and surplus
  - Profits = Total revenue-**total cost**. See Figure 5.
    * Can be negative, zero or positive

Figure 3: SRSC



        * It takes into accout the fixed cost.

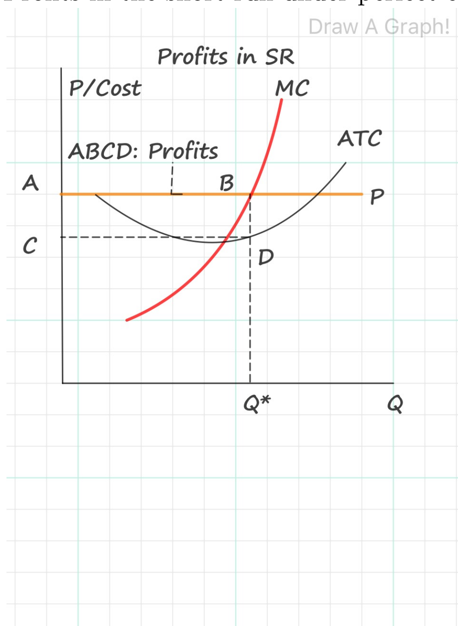
  - Surplus, by definition, is the difference between what you are paid (the price) and the minimum price at which you are willing to accept (MC) for the given quantity. See Figure 5.
    * Equivalently,

$$\text{Surplus} = \text{Total revenue} - \textbf{total variable cost}$$

    * Could be negative, zero or positive.
    * It **does not take into account the fixed cost**.
  - profits = surplus when there is no fixed cost.
  - The key difference between the two: surplus is a **marginal concept**.

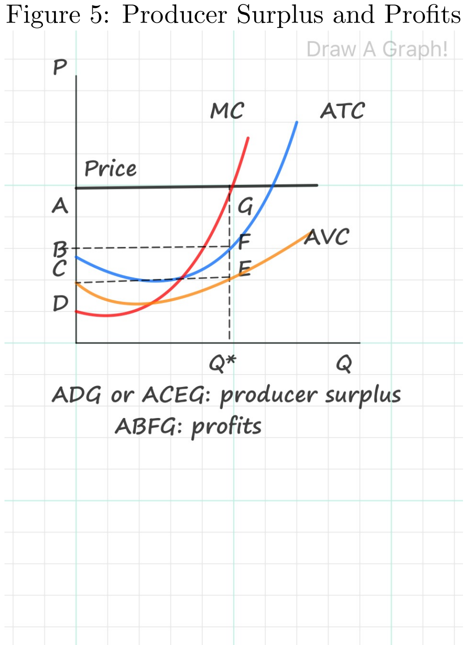
- Long-run
  - At the individual firm level
    * supply curve in the LR:
      · It overlaps with marginal cost curve in the long run
      · The part of LR marginal cost curve that is below LR average **total** cost curve is excluded, i.e. shut down if running loss in the long-run.
  - At the industry level

Figure 4: Profits in the short-run under perfect competition



- ∗ zero profits
  - · Positive short-run profits of individual firms induce entries, driving down the price due to higher supply, untill the price is equal to the long-run average total cost
  - · Price is lowered and lowered until it is equal to the long run average total cost.
  - · In the LR:
$$P = LRATC = LRMC$$
- ∗ Long run supply curve of the industry is horizontal (perfectly elastic). Price changes induces entries and exits and it will drive the price equal to long-run average total cost no matter what.

Figure 5: Producer Surplus and Profits



Session 11. Monopoly

Tao Wang

November 19, 2020

1 The major source of monopolistic power

- Entry barriers.
  - Natural monopoly: the extreme scale of economies, i.e. the long-run average cost decreases as quantity increases in the industry. More efficient to have one producer to produce everything.
  - Switching cost on the consumer's side, i.e. hard to return to the Microsoft once you use Apple;
    - Network goods: Visa/Mastercard; Online social apps such as Facebook;
      - Is Uber a monopoly?
- Product differentiation: imperfect substitution across goods
- Long-run cost advantage via contracts or control of key inputs
- Government regulation, i.e. patent, license.
  - East Indian company. In 1600, Queen Elizabeth I granted a charter to a group of London merchants for exclusive overseas trading rights with the East Indies.
  - Oil industry in Opec countries.

2 Key difference between perfect competition and monopoly

- The monopoly is faced with the **downward** sloping industry demand curve, while a perfectly competitive firm is faced with a **horizontal** demand curve in the market.
- The price prevailing in the market **depends** on the quantity of the monopoly produces in monopoly. No longer a price taker.

- Marginal revenue is no longer equal to the price:

$$MR \neq P$$

$$MR = \underbrace{P}_{PriceEffect} + \underbrace{\frac{\Delta P}{\Delta Q} Q}_{QuantityEffect}$$

  - The first effect is simple: one more unit of good you sell, the revenue increases by the price of the good. This effect is present in perfectly competitive market.
  - The second effect. For a monopoly, in order to sell each additional unit of the good, he/she needs to lower the price. ($\Delta P/\Delta Q$ is negative). So the increase in quantity reduces the price. This effect is only present in monopoly.
  - Since the second term is always negative, we now would expect the marginal revenue curve is always lower than the market demand curve.
  - Profits maximization of the firm now requires

    $$MR = MC$$

    instead of

    $$P = MC$$

  - Derive the monopoly price and quantity in the following example: Demand curve

    $$Q^d = 100 - 10P$$

    MC curve (increasing in Q or constant)

    $$MC = 5 + 0.2Q$$

    Step 1. write the marginal revenue function as a function of Q.
    Inverse demand

    $$P = 10 - 0.1Q^d$$

    $$\rightarrow MR = P + \frac{\Delta P}{\Delta Q} Q = \underbrace{10 - 0.1Q^d}_{P} \underbrace{-0.1}_{\frac{\Delta P}{\Delta Q}} Q^d = 10 - 0.2Q^d$$

    Step 2. solve $Q*$ by the monopoly.

    $$MR = MC$$

    $$\rightarrow Q^*$$

Step 3. solve P^* via demand equation.

$$Q^* = Q^d$$

$$\to P^*$$

- Monopoly produces **less** than in the perfect competition and charge **higher** price than its marginal cost.
- Producer (consumer) surplus is higher (lower) than in the perfect competition.
- Positive deadweight loss. Some consumers are willing to pay for a slightly lower price than P^* which is also higher than the marginal cost, but the monopoly does not produce for that demand.

3 Some scenario analysis

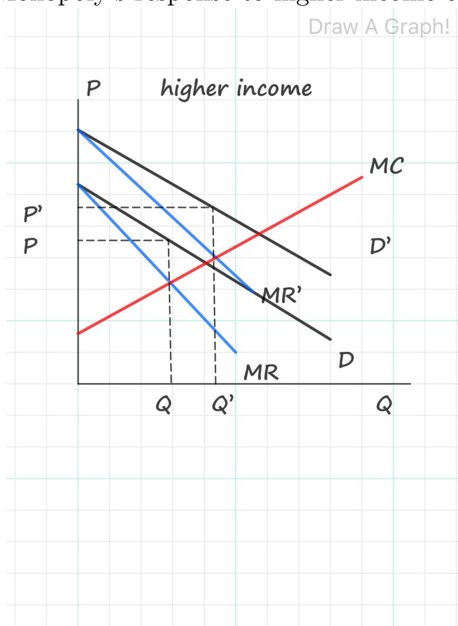
Discuss the effect of the following changes on the market outcome. Specifically, indicate the changes in the following variables.

- Elasticity of demad: E
- Price P^*
- Quantity Q^*
- Producer surplus: PS
- Consumer surplus: CS
- Deadweight loss: DWL

1. Higher income of consumers (See Figure 1)
   - Outward demand shift of the demand. If the shift is parallel, E stays the same [1]. $P \uparrow$, $Q \uparrow$, $PS \uparrow$, CS and DWL could either increase or decrease.
2. A new product introduced that is highly substitutable to the existing good (See Figure 2)
   - The demand for the good becomes more price-elastic, a flatter demand curve, as well as the marginal revenue curve.
   - $E \uparrow$, $P^* \downarrow$, $Q^* \uparrow$, $PS^* \downarrow$, CS^* and DWL could increase or decrease.
3. Higher labor cost (See Figure 3).

[1] More sophisticatedly: E may increase (or decrease) if the good is luxuray or necessity

Figure 1: Monopoly's response to higher income of consumers



- Marginal cost curve has an upward shift.
- E stays the same, $P^* \uparrow$, $Q^* \downarrow$, $CS^* \downarrow$, PS and DWL could either increase or decrease.

Figure 2: Monopoly's response to higher demand elasticity

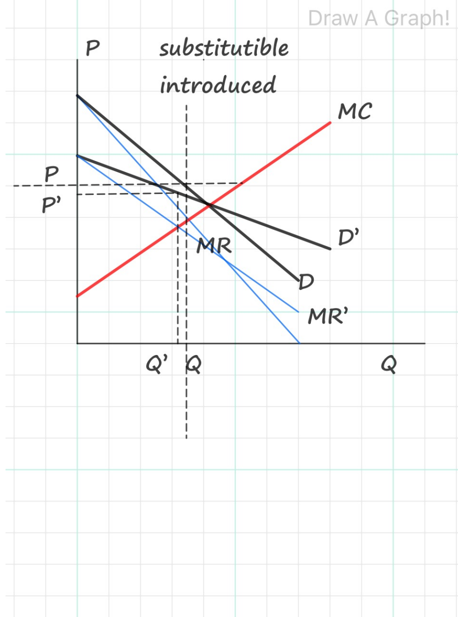
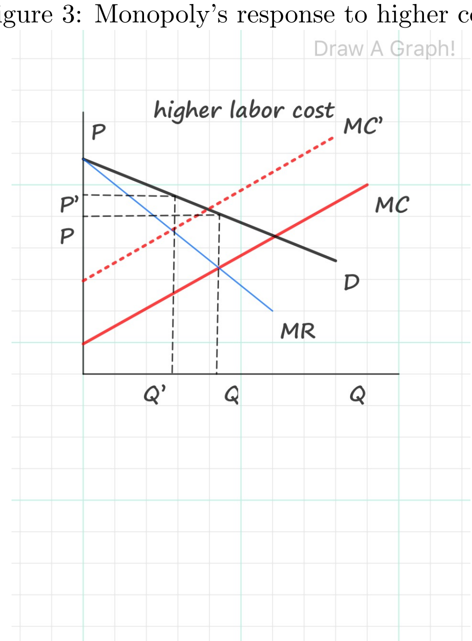


Figure 3: Monopoly's response to higher cost



Session 12. Price Discrimination

Tao Wang

December 3, 2020

1 Price discrimination

- Why do we focus on firms with market power? Only firms with market power have control over the price. A perfectly competitive firm takes the price as given. So there is no need for the pricing strategy.
- What does price discrimination mean?
  - Charging **different consumers** different prices for the **same** product.
    - Different consumers in what?
      - Demand curves: different demand elasticities, different marginal utilities from consuming the same good, etc.
  - Different prices: different unit price.
  - The firm is able to extract some additional producer surplus from consumers.
  - Discrimination is one of the most important concepts in economics. Consumers differ in their income and preferences thus willingness to buy things. In some sense, market mechanisms operate via differentiating these people by inducing their respective choices. More broadly speaking, economics is about resource allocation under scarcity. It is about who gets what and why. This is a question that inherently involves discriminating people by certain criteria based on which the allocation takes place.
- Necessary conditions for price discrimination.
  - Market power: the firm can set the price and it affects the quantity.
  - No arbitrage across consumers who are charged different prices
    - If no, then the single price charged by the monopoly at MC = MR.
  - Direct and indirect price discrimination.

    * Direct (indirect) when the firm knows (does not know) the demand curve of the consumer before the transaction.
        · Set price for the consumer x so that MR from selling the good to x equal to the MC of the firm.
    * Direct (indirect) when the firm can (cannot) charge the prices differently toward consumers based on observable characteristics.

– Examples
    * Uber shows a higher price to you than to your friend at the same location for the same trip at 10 pm because Uber knows you go out at this time every week.
    * The train to Machu Picchu is \$20 for foreign tourists but only \$2 for local people. (Passports are required for the ticket purchase and the ticket has a name printed on it.)
    * "Farang price": higher price for white foreign tourists in Thailand.
    * The prices of the hotel that pops up are different depending on which language you use to search on Google.
    * Free entry for girls and \$10 for guys in nightclubs.
    * Black Friday sales.
    * DC metro charges an extra dollar for the new metro card.
    * Students or elder discounts.
    * Different premiums of life insurance for people at a different age.
    * Amazon homepage shows different prices of the same deal to a frequent buyer and a first-time visitor.
    * Starbucks: three different cup sizes.

Figure 1: Price discrimination

Session 13. Imperfect Competition

Tao Wang

December 21, 2020

1 Different market structure

Perfect competition and monopoly are two extremes of the market structure. There are market structures that are in between the two extremes but are actually a more common representation of the real world.

In the order from perfection to imperfection: market could take the following forms

- Perfect competition
  - Firm is the price taker
  - Infinite number of firms
  - Perfectly elastic demand (facing the individual firms)
  - No strategic interaction between firms, i.e. A's best choice does not depend on B's choice.
  - Zero profits in the long-run
  - P=LRAC= MC
- Monopolistic competition
  - Firm is not price taker
  - A large number of firms
  - Goods are differentiated
  - Inelastic demand facing individual firms
  - Free entry/barrier
  - No strategic interaction
  - Zero profit in the long-run
  - P = LRAC > MC
- Oligopoly
  - Not price taker
  - Small number of firms

- Could be identical or differentiated goods
- Strategic interaction
- No free entry, thus could run positive long-run profits
- Different types
  - If identical goods and firms coordinate with each other: collusion and Cartel
    - Together act as a single monopoly and extract the maximum industry profits.
    - But hard to maintain because individual firms are tempted to deviate.
  - If the firms compete and set the price: Bertrand
    - Different types
      1. If the goods are identical and firm move together: static Bertrand for identical goods;
         (a) P = MC, i.e. firms race the price all the way to marginal cost.
      2. If the goods are identical and some firms move first: dynamic Bertrand/Stackerberg
         (a) The first mover has advantage because you always know how your follower would react to your strategy.
      3. If the goods are differentiated: Bertrand for differentiated good.
         (a) The price of one good affects the demand for the other good.
         (b) One firm's price decision changes the decision of the other firm.
  - If the goods are identical and firms compete and set the quantity: Cournot
    - There is a single price in the market
    - The price is determined by the total quantity

- Monpoly
  - Not price taker
  - Single firm
  - Inelastic demand facing the firm (or industry, they are the same)
  - No free entry.
  - No strategic interaction (because there is only monopoly)
  - P > MC (mark-up) and P >LRAC (positive profits)

2 Equilibrium when strategic interaction is involved

- The new equilibrium concept.

  Previously, market equilibrium simply means market clearing. Demand = Supply.

  When the strategic interaction is involved: equilbrium is the Nash Equilibrium. It means

  1. Market clearing
  2. No person would deviate from the current strategies given the other's strategies.
     - Everyone has their best responses taking the others' as given.
     - Everyone recognizes that his/her strategy will affect others' strategy in the equilibrium.

- How to find Nash Equilibrium.
  1. Find the reaction function of each individual taking others' choice as given. The reaction function will be a function of your choice variable and others' choice.
  2. Jointly solve everyone's reaction by let all reaction functions coincide.

3 Sovling Problems in Different Market Structures: Examples

1. Cournot with identical goods
   - Question: the market demand curve is Q = 90-3P for the good. There are two firms A and B engaging in a Cournot competition. The market quantity is the sum of two firms's quantities: $Q = q_a + q_b$. The marginal cost of a and b are, 0 and 2, respectively. Find the equilibrium price P and quantity q_a and q_b in the equilibrium.
   - Answer
     (a) Ask yourself: what is the variable which the firm chooses to achieve its goal? Price or quantity? (The reason we want to ask this is that we want to write MR and MC as a function of the choice variable)
       - Quantity, since it is Cournot. Firm a chooses q_a and b chooses q_b.

     (b) Write MR_a as a function of q_a (your choice variable) and q_b (the other player's choice).

- $TR_a = q_a P$ (Since we want to write TR_a as a function of q_a, we should replace P with q_a in it.) This gives

$$TR_a = q_a \underbrace{(30 - \frac{Q}{3})}_{P} = 30q_a - \frac{q_a^2 + q_a q_b}{3}$$

Now, we want to take derivative of total revenue with respect to the variable you **choose**, i.e. the q_a.

$$MR_a = 30 - \frac{q_b}{3} - \frac{2q_a}{3}$$

(c) Let MR_a equal to marginal cost of firm a, we could obtain the reaction function of a.

$$\begin{aligned} MR_a &= 0 \\ \rightarrow 30 - \frac{q_b}{3} - \frac{2q_a}{3} &= 0 \\ \rightarrow q_a &= 45 - 0.5q_b \end{aligned}$$

This is the reaction function of firm a, as a function of other's choice.

(d) By the same token, we can get the reaction function of b is

$$\rightarrow q_b = 42 - 0.5q_a$$

(e) Combing the two reaction functions above and we can solve q_a and q_b. Substituting one into the other, we get

$$q_a = 32$$

and

$$q_b = 26$$

(f) Total quantity $Q = 32 + 26 = 58$. Plugging it in the market demand curve, we get the price in the equilibrium.

$$P \approx 10$$

2. Bertrand with differentiated goods

- Question: two firms a and b produce differentiated goods and engage in a Bertrand competition by choosing the price of their respective good. Individual demand curve for good a and b are respectively $q_a = 900 - 2p_a + p_b$ and $q_b = 900 - 2p_b + p_a$. (notice here the demand for either good is affected by the price of the competitor's good.) The marginal cost of a and b are 0. Find the price of a and b and quantity of the both goods a and b in the market.

- Answer
  (a) Again, ask yourself: what is the choice variable by the firms here?
    - Price, instead of quantity (different from the previous example). This is a Bertrand competition. Firm a (b) chooses p_a (p_b). Then it will be more convenient to write the reaction function as a function of p_a and p_b.

  (b) To write the MR_a as a function of p_a, first, work with the total revenue.

$$TR_a = q_a p_a = p_a(900 - 2p_a + p_b) = 900p_a - 2p_a^2 + p_a p_b$$

  We obtain the marginal revenue by taking the derivative of TR_a with respect to the choice variable q_a here. **Be careful here that** the marginal revenue is not the change in total revenue from change in quantity but in price. This is because our choice variable is now price instead of quantity.

$$MR_a = 900 - 4p_a + p_b$$

  (c) Letting MR_a equal to marginal cost 0 gives the reaction function of a as a function of p_b.

$$p_a = 225 + 0.25p_b$$

  (d) By the same token, we can solve the reaction function of b being

$$p_b = 225 + 0.25p_a$$

  (e) Combining the two reaction functions, we can solve p_a and p_b in the equilbrium.

$$p_b = 300$$

  and

$$p_a = 300$$